An Approach to Observer-Based Decentralized Control

under Periodic Protocols

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Where innovation starts

ACC, Baltimore - WeC16.3
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Motivation

Interest in control of large-scale systems that are **physically distributed over a wide area:**

- Observer-based
- Decentralized
- Wireless $\rightarrow$ Network Effects

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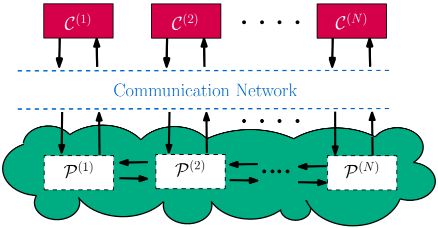
Problem Description

We consider decentralized control ***design*** and ***stability*** analysis for

- large-scale continuous-time linear plant
- via a multi-purpose network with
  - communication constraints: not all outputs and inputs can be communicated simultaneously
  - uncertain time-varying transmission intervals $h_k \in [\underline{h}, \overline{h}]\ \forall k \in \mathbb{N}$



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Outline

NCS Model
- Plant Description
- Plant Decomposition
- Communication Constraints
- Closed Loop Model

Stability

Design

Numerical Example

Conclusions

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Model - Plant

The plant is given by

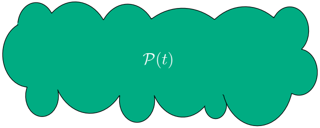
$$\mathcal{P}(t) := \begin{cases} \dot{x}(t) &= Ax(t) + B(t)\hat{u}(t) \\ y(t) &= Cx(t) \end{cases}$$

We can express this system with time-varying transmission intervals in the following way

$$\mathcal{P}_{h_k} := \begin{cases} x_{k+1} &= \bar{A}_{h_k} x_k + \bar{B}_{h_k} \hat{u}_k \\ y_k &= Cx_k \end{cases}, \quad h_k \in [\underline{h}, \overline{h}]$$

where $\bar{A}_{h_k} := e^{A h_k}$, $\bar{B}_{h_k} := \int_0^{h_k} e^{As} ds B$

Model - Decomposition



Decomposition

$$\mathscr{P}(t) := \begin{cases} \dot{x}(t) & = & Ax(t) + B\hat{u}(t) \\ y(t) & = & Cx(t) \end{cases}$$

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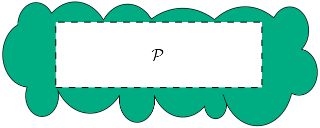
Model - Decomposition



Decomposition

$$\mathcal{P}(t) := \begin{cases} \dot{x}(t) &= Ax(t) + B\hat{u}(t) \\ y(t) &= Cx(t) \end{cases}$$

$$\downarrow$$

$$\mathcal{P} := \begin{cases} x_{k+1} &= \bar{A}x_k + \bar{B}\hat{u}_k \\ y_k &= Cx_k \end{cases}$$

$$\bar{A} = e^{Ah_\star}, \bar{B} = \int_0^{h_\star} e^{As} ds B$$

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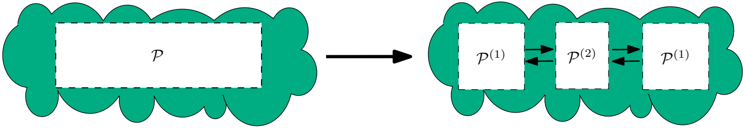


Subsystems
$i \in \{1, ..., N\}$

Decomposition

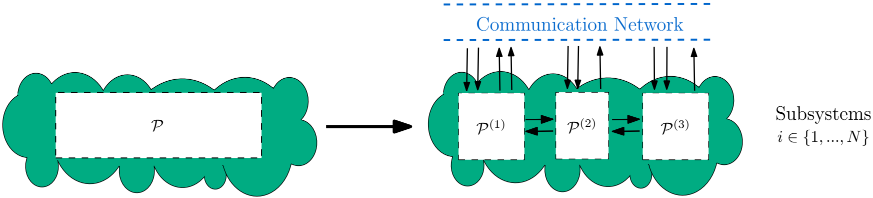
$$\mathcal{P}(t) := \begin{cases} \dot{x}(t) &= Ax(t) + B\hat{u}(t) \\ y(t) &= Cx(t) \end{cases}$$

$$\downarrow$$

$$\mathcal{P} := \begin{cases} x_{k+1} &= \bar{A}x_k + \bar{B}\hat{u}_k \\ y_k &= Cx_k \end{cases} \quad \rightarrow \mathcal{P}^{(i)} := \begin{cases} z_{k+1}^{(i)} &= \bar{A}_i z_k^{(i)} + \bar{B}_i \hat{u}_k^{(i)} \\ & \quad + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(\bar{A}_{i,j} z_k^{(j)} + \bar{B}_{i,j} \hat{u}_k^{(j)} \right) \\ y_k^{(i)} &= \bar{C}_i z_k^{(i)} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{C}_{i,j} z_k^{(j)} \end{cases}$$

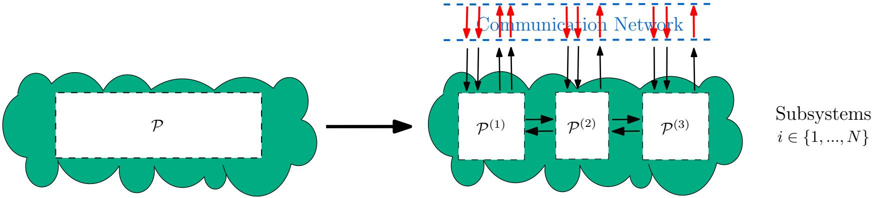
$$\bar{A} = e^{Ah_\star}, \ \bar{B} = \int_0^{h_\star} e^{As} ds B$$

Model - Network Effects



Communication Constraints

Model - Network Effects



Communication Constraints

Model - Network Effects

Communication Constraints

Only one node is allowed to transmit information at each transmission time

Node - A collection of sensors and/or actuators are allowed communicate over a network simultaneously

Periodic Protocol - grant network access to each node in a periodic fashion

$\sigma_k \in \{1, 2, ..., n_T\}$ denotes the node that has access at transmission time $k \in \mathbb{N}$

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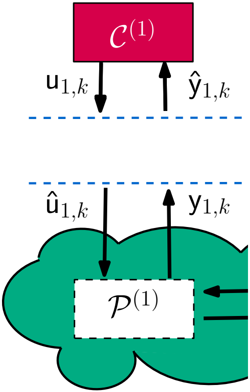
Model - Network Effects

Communication Constraints

$$\hat{u}_{j,k} = \begin{cases} u_{j,k} & \text{if node } j \text{ has access} \\ \hat{u}_{j,k-1} & \text{otherwise} \end{cases}$$



Model - Network Effects

Communication Constraints

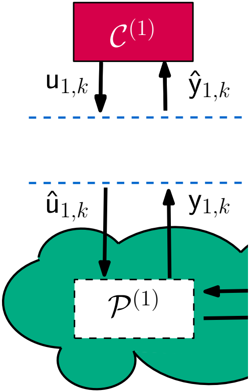
$$\hat{u}_{j,k} = \begin{cases} u_{j,k} & \text{if node } j \text{ has access} \\ \hat{u}_{j,k-1} & \text{otherwise} \end{cases}$$

Mathematically we express this as

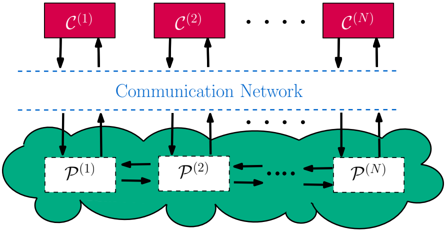
$$\begin{bmatrix} \hat{u}_k \\ \hat{y}_k \end{bmatrix} = \Gamma_{\sigma_k} \begin{bmatrix} u_k \\ y_k \end{bmatrix} + (I - \Gamma_{\sigma_k}) \begin{bmatrix} \hat{u}_{k-1} \\ \hat{y}_{k-1} \end{bmatrix}$$

where $\Gamma_{\sigma_k} = diag(\gamma_{j,\sigma_k})$

$$\gamma_{i,\sigma_k} = \begin{cases} 1 & \text{if } u_{j,k}/y_{j,k} \text{ has network access} \\ 0 & \text{otherwise} \end{cases}$$



Controller



$$\mathcal{C}^{(i)}_{\sigma_k} := \begin{cases} \tilde{x}^{(i)}_{k+1} = A_i \tilde{x}^{(i)}_k + B_i \hat{u}^{(i)}_k + L_{i,\sigma_k} \Gamma^{y}_{i,\sigma_k} (\hat{y}^{(i)}_k - \bar{C}_i \tilde{x}^{(i)}_k) \\ u^{(i)}_k = -K_{i,\sigma_k} \tilde{x}^{(i)}_k \end{cases}$$

- discrete-time
- observer-based
- decentralized
- switch based on protocol

Model - Summary

Plant Dynamics:

$$\mathcal{P}_{h_k} := \begin{cases} z_{k+1} &= \bar{A}_{h_k} z_k + \bar{B}_{h_k} \hat{u}_k \\ y_k &= \bar{C} z_k \end{cases} \qquad h_k \in [\underline{h}, \overline{h}]$$

where

$$\begin{cases} \hat{u}_k &= \Gamma^u_{\sigma_k} u_k + (I - \Gamma^u_{\sigma_k}) \hat{u}_{k-1} \\ \hat{y}_k &= \Gamma^y_{\sigma_k} y_k + (I - \Gamma^y_{\sigma_k}) \hat{y}_{k-1} \end{cases} \qquad \sigma_k \in \{1, \ldots, n_T\}$$

Controller Dynamics:

$$\mathcal{C}^{(i)}_{\sigma_k} := \begin{cases} \tilde{x}^{(i)}_{k+1} &= A_i \tilde{x}^{(i)}_k + B_i \hat{u}^{(i)}_k + L_{i,\sigma_k} \Gamma^y_{i,\sigma_k} (\hat{y}^{(i)}_k - \bar{C}_i \tilde{x}^{(i)}_k) \\ u^{(i)}_k &= -K_{i,\sigma_k} \tilde{x}^{(i)}_k \end{cases}$$

Model - Closed Loop

The closed loop model can be written as a discrete-time switched system with exponential uncertainty:

$$\bar{x}_{k+1} = \tilde{A}_{c,h_k,\sigma_k}\bar{x}_k, \quad h_k \in [\underline{h}, \bar{h}], \quad \sigma_k \in \{1, ..., N\}$$

where

$$\bar{x}_k = \begin{bmatrix} \eta_k \\ z_k \\ e^u_k \\ e^y_k \end{bmatrix} = \begin{bmatrix} \tilde{z}_k - z_k \\ z_k \\ \hat{u}_{k-1} - u_k \\ \hat{y}_{k-1} - y_k \end{bmatrix}$$

and

$$\tilde{A}_{c,h_k,\sigma_k} =$$

$$\begin{bmatrix} A_d - L_{\sigma_k}\Gamma^y_{\sigma_k}C_d + \Delta B_{c,h_k}K_{\sigma_k} & L_{\sigma_k}\Gamma^y_{\sigma_k}C_c - \Delta A_{c,h_k} + \Delta B_{c,h_k}K_{\sigma_k} & -\Delta B_{c,h_k}(I - \Gamma^u_{\sigma_k}) & 0 \\ -\bar{B}_{h_k}K_{\sigma_k} & \bar{A}_{h_k} - \bar{B}_{h_k}K_{\sigma_k} & \bar{B}_{h_k}(I - \Gamma^u_{\sigma_k}) & 0 \\ K_{\sigma_k}(A_d - L_{\sigma_k}\Gamma^y_{\sigma_k}C_d - B_dK_{\sigma_k} - I) & K_{\sigma_k}(A_d + L_{\sigma_k}\Gamma^y_{\sigma_k}C_c - B_dK_{\sigma_k} - I) & (K_{\sigma_k}B_d + I)(I - \Gamma^u_{\sigma_k}) & 0 \\ \bar{C}\bar{B}_{h_k}K_{\sigma_k} & \bar{C}(I - \bar{A}_{h_k} + \bar{B}_{h_k}K_{\sigma_k}) & -\bar{C}\bar{B}_{h_k}(I - \Gamma^u_{\sigma_k}) & I - \Gamma^y_{\sigma_k} \end{bmatrix}$$

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Stability

Due to the transmission variance $h_k \in [\underline{h}, \overline{h}]\ \forall k \in \mathbb{N}$, there is an infinite amount of sequences to check for stability

$$\left\{\tilde{A}_{c,h,\sigma} \mid h \in [\underline{h}, \overline{h}]\right\}$$

Stability

Due to the transmission variance $h_k \in [\underline{h}, \overline{h}]\ \forall k \in \mathbb{N}$, there is an infinite amount of sequences to check for stability

$$\left\{\tilde{A}_{c,h,\sigma} \mid h \in [\underline{h}, \overline{h}]\right\} \subseteq \left\{\sum_{j=1}^{M} \alpha^j \left(F_{\sigma,j} + G_j \Delta H_\sigma\right)\right\}$$

Therefore we prove stability on an overapproximation of the original model, which is achieved by

(i) gridding a finite number of points in $[\underline{h}, \overline{h}]$
(ii) adding norm-bounded uncertainty to each grid point to capture the non-linearity between grid points.

Using the candidate $V_{\sigma_k}(x_k) = \bar{x}_k^\top P_{\sigma_k} \bar{x}_k$ along with the full-block S-procedure, a finite set of LMIs can be derived on the overapproximation for periodic protocols.

Design

Design Problem:

Given a decomposition and a protocol, how to choose L_{σ_k} and K_{σ_k} such that the closed-loop NCS is stable?

Goal:

Provide LMI conditions to design L_{σ_k} and K_{σ_k} using the Lyapunov canidate

$$V_{\sigma_k}(x_k) = \bar{x}_k^\top P_{\sigma_k} \bar{x}_k \geq 0$$

$$\Delta V_{\sigma_k}(x_k) = \tilde{A}_{c,h_k,\sigma_k}^\top P_{\sigma_{k+1}} \tilde{A}_{c,h_k,\sigma_k} - P_{\sigma_k} \prec 0$$

we use the fact that σ_k is a known periodic function

Design

Our approach:

1. Simplify the design problem in the following way:
    - constant transmission intervals
    - ignore subsystem coupling

   $\rightarrow$ design considering only the protocol σ_k
2. Verify stability of the model including varying transmission intervals and subsystem coupling by using overapproximation technique

Design

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Entire closed loop system matrix

$$\tilde{A}_{c,h_k,\sigma_k} =$$

$$\begin{bmatrix} A_d - L_{\sigma_k}\Gamma^y_{\sigma_k}C_d + \Delta B_{c,h_k}K_{\sigma_k} & L_{\sigma_k}\Gamma^y_{\sigma_k}C_c - \Delta A_{c,h_k} + \Delta B_{c,h_k}K_{\sigma_k} & -\Delta B_{c,h_k}(I - \Gamma^u_{\sigma_k}) & 0 \\ -\bar{B}_{h_k}K_{\sigma_k} & \bar{A}_{h_k} - \bar{B}_{h_k}K_{\sigma_k} & \bar{B}_{h_k}(I - \Gamma^u_{\sigma_k}) & 0 \\ K_{\sigma_k}(A_d - L_{\sigma_k}\Gamma^y_{\sigma_k}C_d - B_d K_{\sigma_k} - I) & K_{\sigma_k}(A_d + L_{\sigma_k}\Gamma^y_{\sigma_k}C_c - B_d K_{\sigma_k} - I) & (K_{\sigma_k}B_d + I)(I - \Gamma^u_{\sigma_k}) & 0 \\ \bar{C}\bar{B}_{h_k}K_{\sigma_k} & \bar{C}(I - \bar{A}_{h_k} + \bar{B}_{h_k}K_{\sigma_k}) & -\bar{C}\bar{B}_{h_k}(I - \Gamma^u_{\sigma_k}) & I - \Gamma^y_{\sigma_k} \end{bmatrix}$$

Design

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   → design considering only the protocol σ_k
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With design simplifications

$$\tilde{A}_{\sigma_k} = \left[\begin{array}{c|cc|c} A_d - L_{\sigma_k}\Gamma^y_{\sigma_k} C_d & 0 & 0 & 0 \\ \hline -B_d K_{\sigma_k} & A_d - B_d K_{\sigma_k} & B_d(I - \Gamma^u_{\sigma_k}) & 0 \\ K_{\sigma_k}(A_d - L_{\sigma_k}\Gamma^y_{\sigma_k} C_d - B_d K_{\sigma_k} - I) & K_{\sigma_k}(A_d - B_d K_{\sigma_k} - I) & (K_{\sigma_k} B_d + I)(I - \Gamma^u_{\sigma_k}) & 0 \\ \hline C_d B_d K_{\sigma_k} & C_d(I - A_d + B_d K_{\sigma_k}) & -C_d B_d(I - \Gamma^u_{\sigma_k}) & I - \Gamma^y_{\sigma_k} \end{array}\right]$$

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Design

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Interesting special case: Controllers are hard wired ($\Gamma^u_{\sigma_k} = I$) we have

$$\tilde{A}_{\sigma_k} = \left[\begin{array}{c|c|c} A_d - L_{\sigma_k}\Gamma^y_{\sigma_k}C_d & 0 & 0 \\ \hline -B_d K_{\sigma_k} & A_d - B_d K_{\sigma_k} & 0 \\ \hline C_d B_d K_{\sigma_k} & C_d(I - A_d + B_d K_{\sigma_k}) & I - \Gamma^y_{\sigma_k} \end{array}\right]$$

$\rightarrow$ convex LMI design conditions

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Numerical Example

Plant Model

$$
\left[\begin{array}{c|c} A & B \\ \hline C & \end{array}\right] = \left[\begin{array}{cccc|ccc}
0.6 & -4.2 & 0.1 & 2.1 & 0.7 & 1.9 & -0.02 \\
0.1 & -2.1 & 0.01 & 0 & 0 & 1 & -0.01 \\
0 & 0 & -3.2 & 0.2 & 0 & 0 & 0.8 \\
0 & -0.03 & 5.3 & -0.2 & 0 & 0 & -0.4 \\
\hline
1 & 4 & 0 & 0.05 & & & \\
0.2 & 1 & 0 & 0 & & & \\
0 & 0 & 2 & 0 & & &
\end{array}\right]
$$

Plant Model

$$
\left[\begin{array}{c|c} A & B \\ \hline C & \end{array}\right] = \left[\begin{array}{cc:cc|cc:c}
0.6 & -4.2 & 0.1 & 2.1 & 0.7 & 1.9 & -0.02 \\
0.1 & -2.1 & 0.01 & 0 & 0 & 1 & -0.01 \\
\hdashline
0 & 0 & -3.2 & 0.2 & 0 & 0 & 0.8 \\
0 & -0.03 & 5.3 & -0.2 & 0 & 0 & -0.4 \\
\hline
1 & 4 & 0 & 0.05 & & & \\
0.2 & 1 & 0 & 0 & & & \\
\hdashline
0 & 0 & 2 & 0 & & &
\end{array}\right], \quad h_\star = 1
$$

Numerical Example

Plant Model

$$\left[\begin{array}{c|c} A & B \\ \hline C & \end{array}\right] = \left[\begin{array}{cc|cc|cc|c} 0.6 & -4.2 & 0.1 & 2.1 & 0.7 & 1.9 & -0.02 \\ 0.1 & -2.1 & 0.01 & 0 & 0 & 1 & -0.01 \\ \hline 0 & 0 & -3.2 & 0.2 & 0 & 0 & 0.8 \\ 0 & -0.03 & 5.3 & -0.2 & 0 & 0 & -0.4 \\ \hline 1 & 4 & 0 & 0.05 & & & \\ 0.2 & 1 & 0 & 0 & & & \\ \hline 0 & 0 & 2 & 0 & & & \end{array}\right], \quad h_\star = 1$$

Nodes and Protocol

$$\Gamma_1^y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2^y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma_3^y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_{1,2,3}^u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_k = 1, 2, 3, 1, 2, \ldots$$

Switching Gains

$$L_1 = \begin{bmatrix} 6.24 & -24.89 & 0 \\ -0.73 & 3.46 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.32 & 2.65 & 0 \\ 0.16 & 0.15 & 0 \\ 0 & 0 & 0.28 \\ 0 & 0 & 3.27 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0.57 & 0.44 & 0 \\ 0.04 & 0.02 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$K_l = K = \begin{bmatrix} 1.94 & -1.40 & 0 & 0 \\ -0.56 & -0.86 & 0 & 0 \\ 0 & 0 & 1.36 & 0.81 \end{bmatrix}$$

Now we can verify if the designed gains are stable including subsystem coupling and and varying transmission intervals

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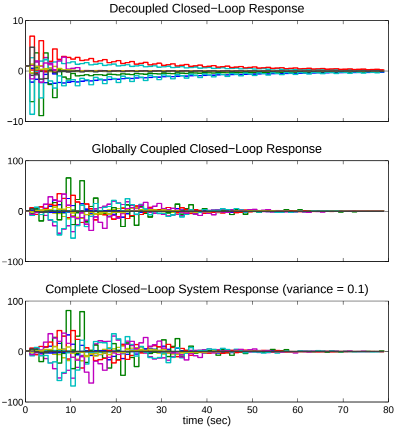
Numerical Example



The system is stable for $h_k \in [0.9, 1.1]$

Conclusions

- We presented a model for an NCS which includes
  - varying transmission intervals $[\underline{h}, \overline{h}]$
  - communication constraints (protocol)

  The controllers are
  - observer-based
  - decentralized
  - switch based on protocol
- Stability can be proven via LMIs based on an overapproximation
- First approach at the design of K_{σ_k} and L_{σ_k} (based on a simplified model)

Future Work:

- Improve design method
- Extend the class of protocols that are able to be analyzed
- Include more NCS effects
- Extend model to include distributed control