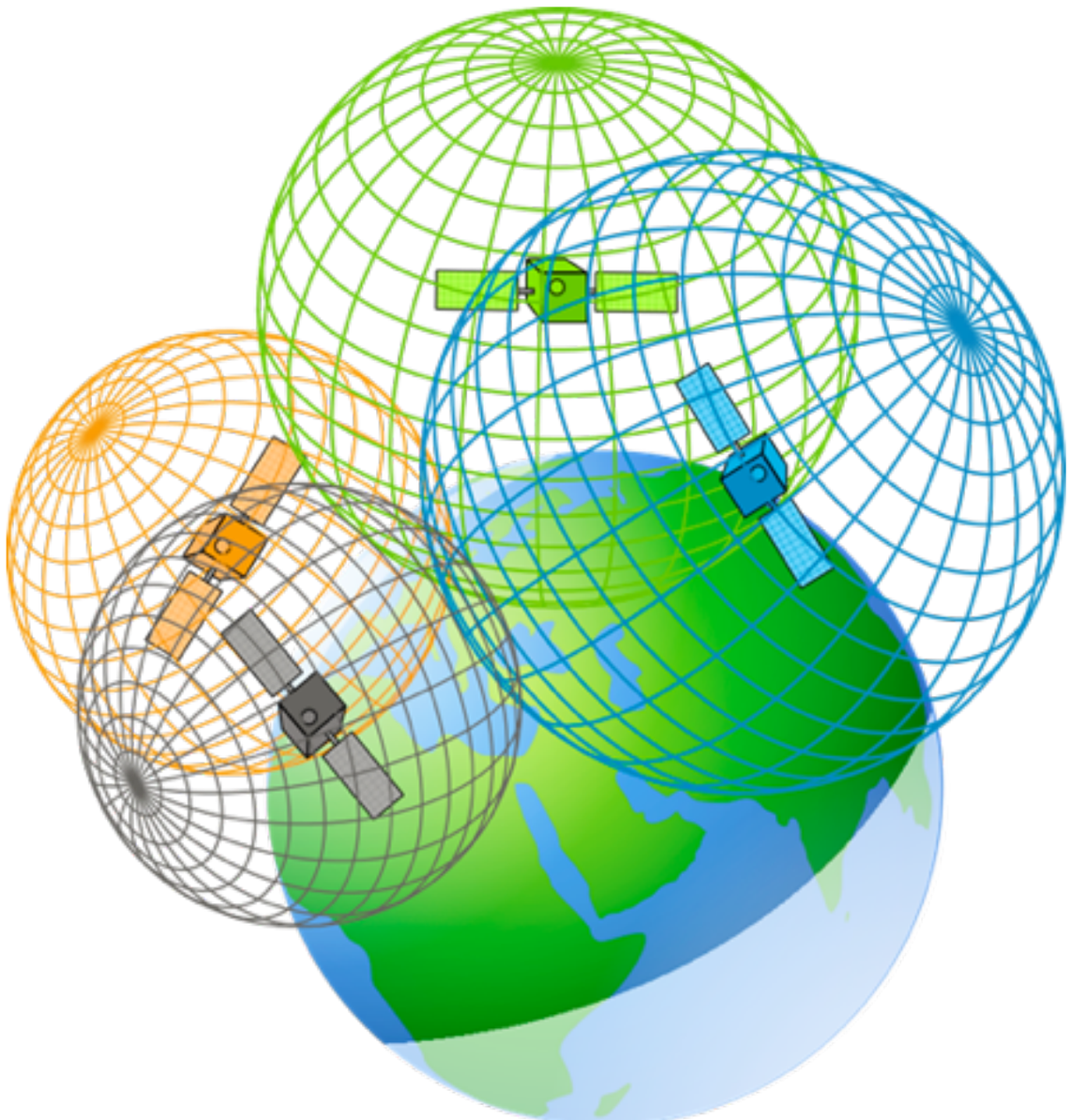
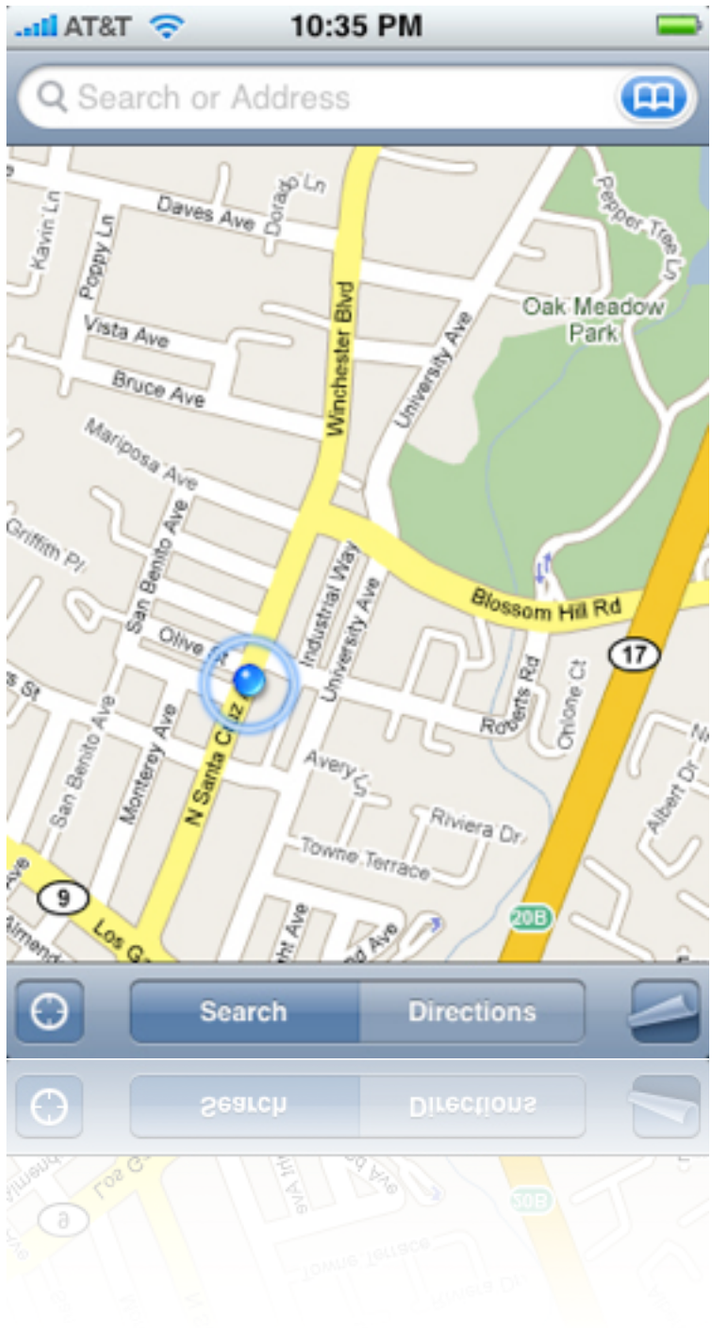
The Geometry of GPS



Nicolas Bauer

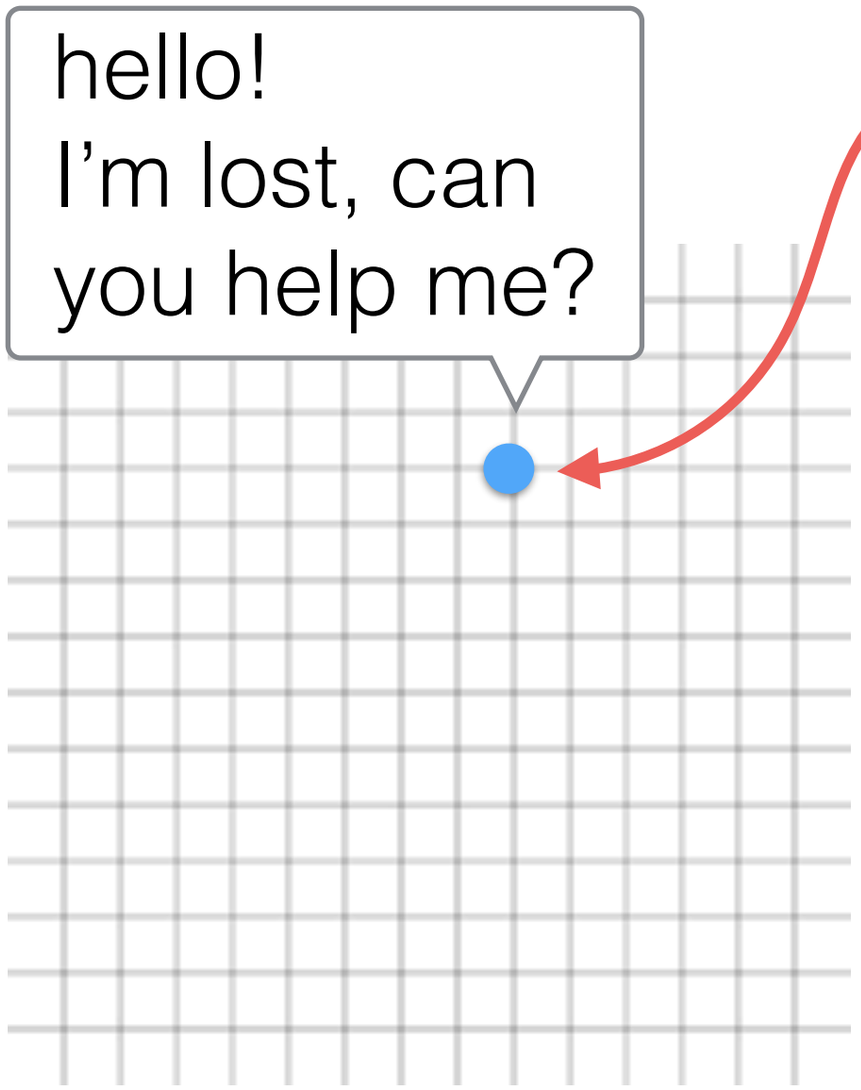
Topic



How does your phone always know where you are?

The problem is finding where you are relative to other things that know where they are.

Localization

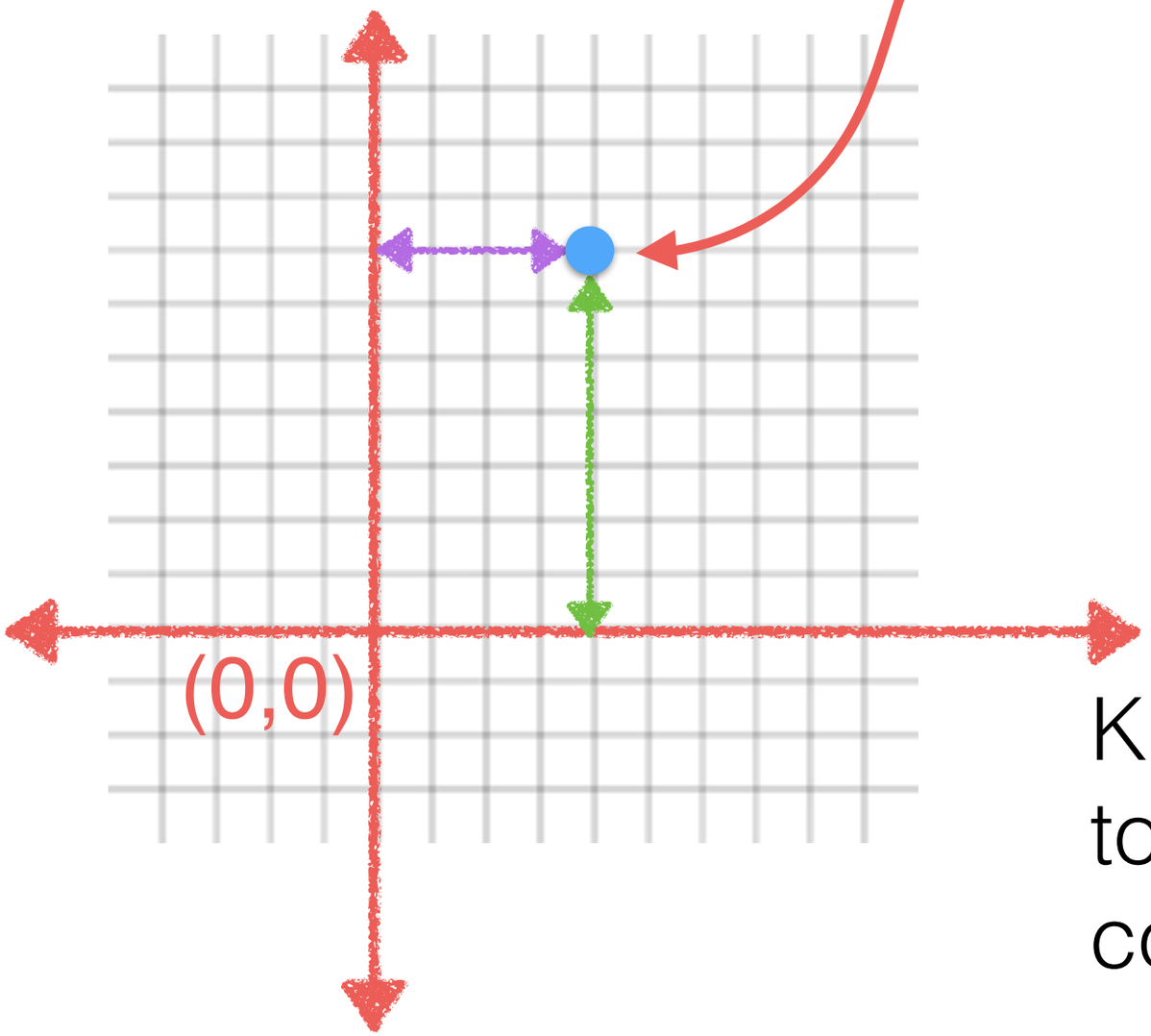


Let's say you are a point.

How do you know your coordinates?

What do you need to know?

Localization



Let's say you are a point.

How do you know your (x,y) coordinates?

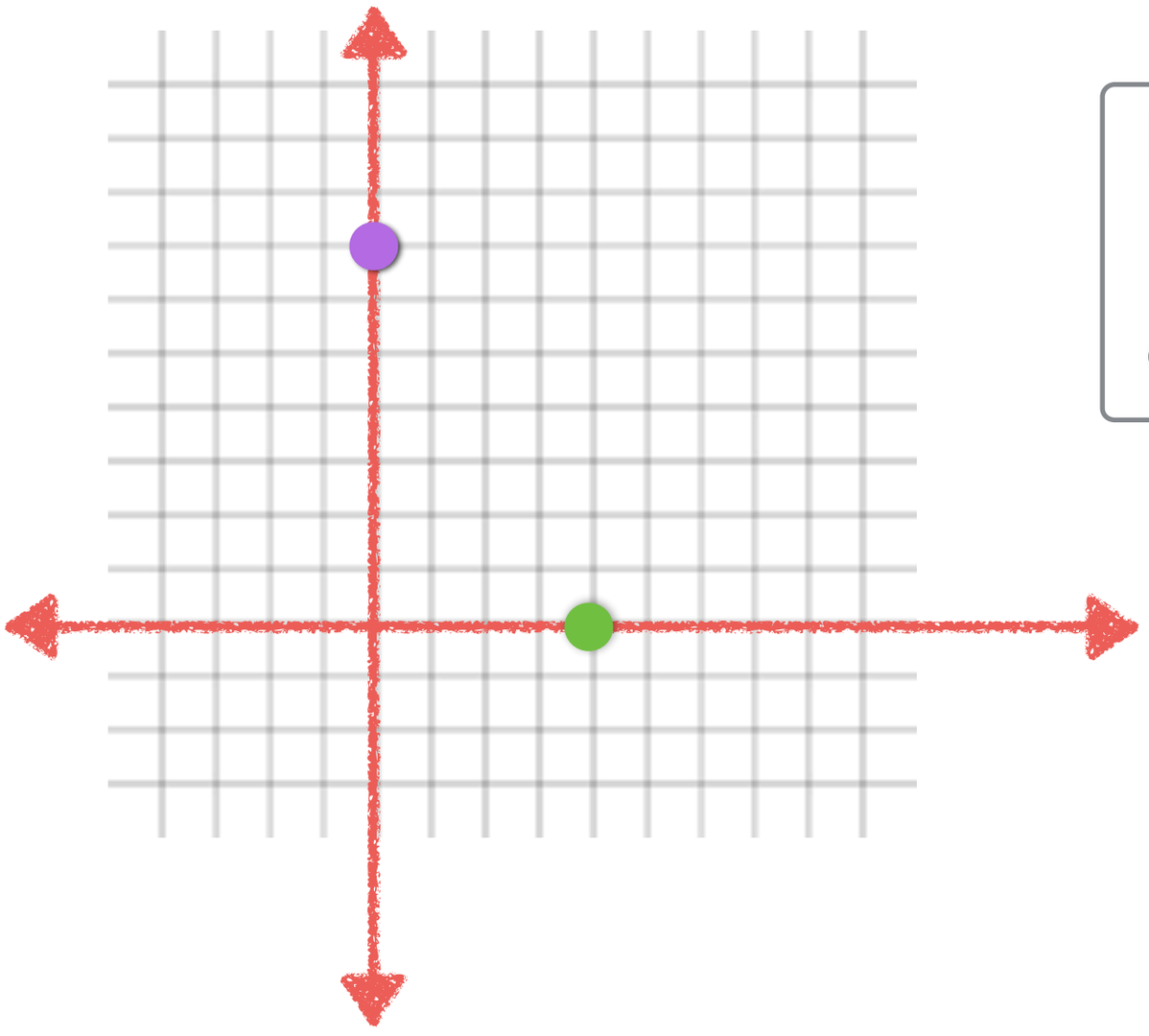
What do you need to know?

Knowing your position relative to (0,0) is enough to determine coordinates.

But what if we didn't know our position relative to (0,0)?

Localization

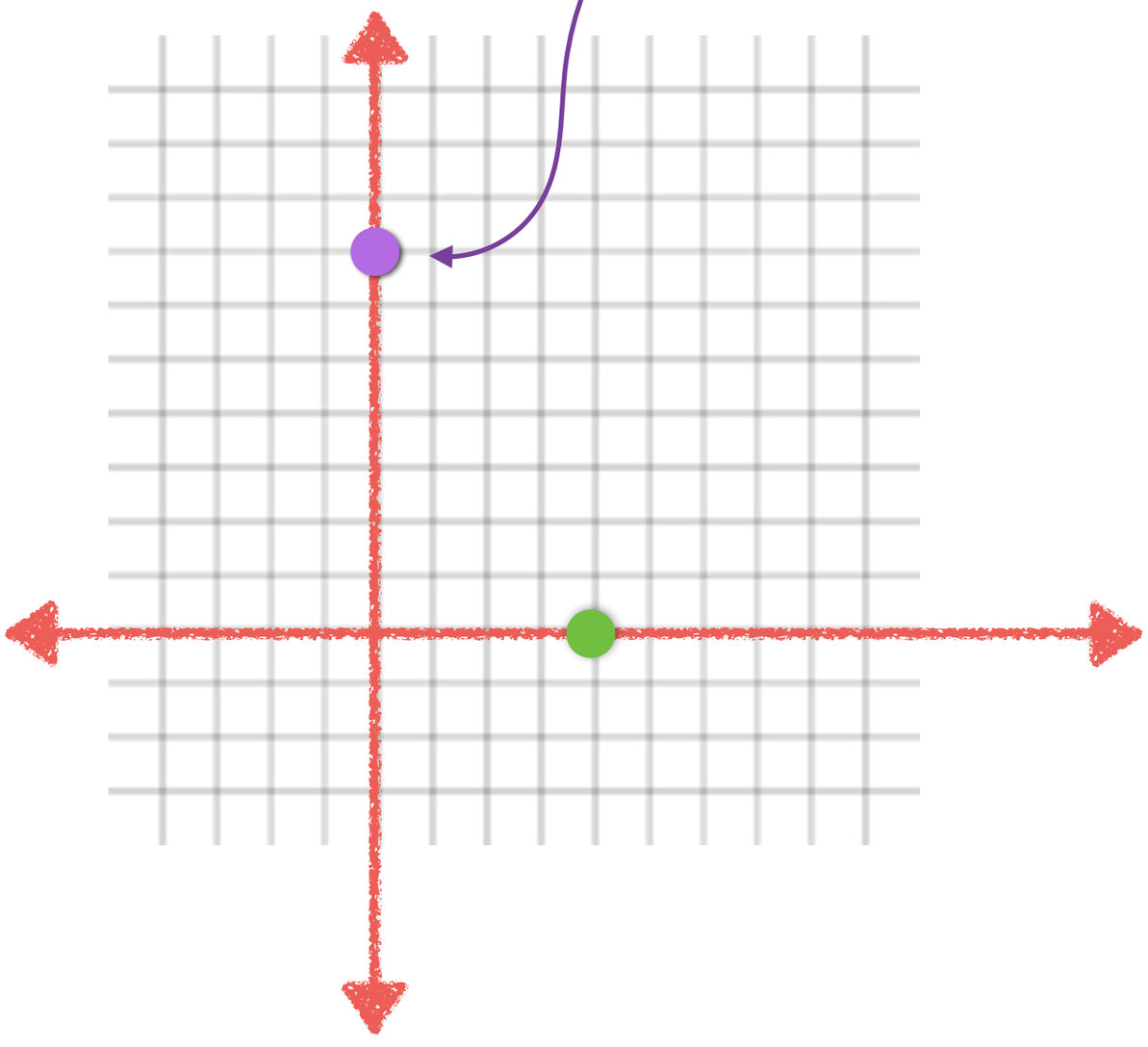
Where could this dot be?



hello!
I'm lost :(
can you find me?

I know I am
- 4 units from (0,7)
- 7 units from (4,0)

Localization



2 possibilities

Distance Formula

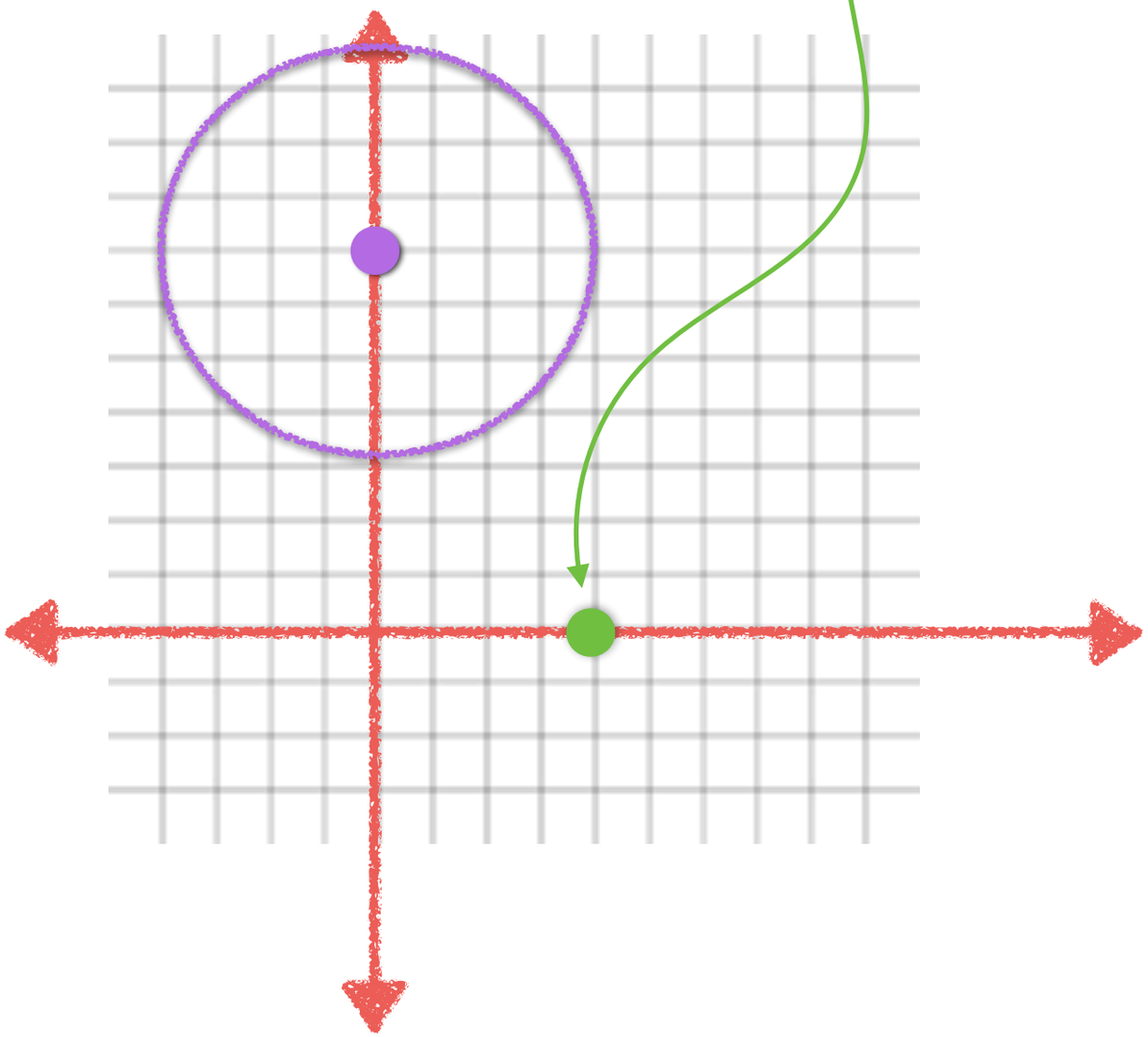
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(x_2, y_2) = (0, 7)$

$(x_1, y_1) = (x, y)$

$d = 4$

Localization



2 possibilities

Distance Formula

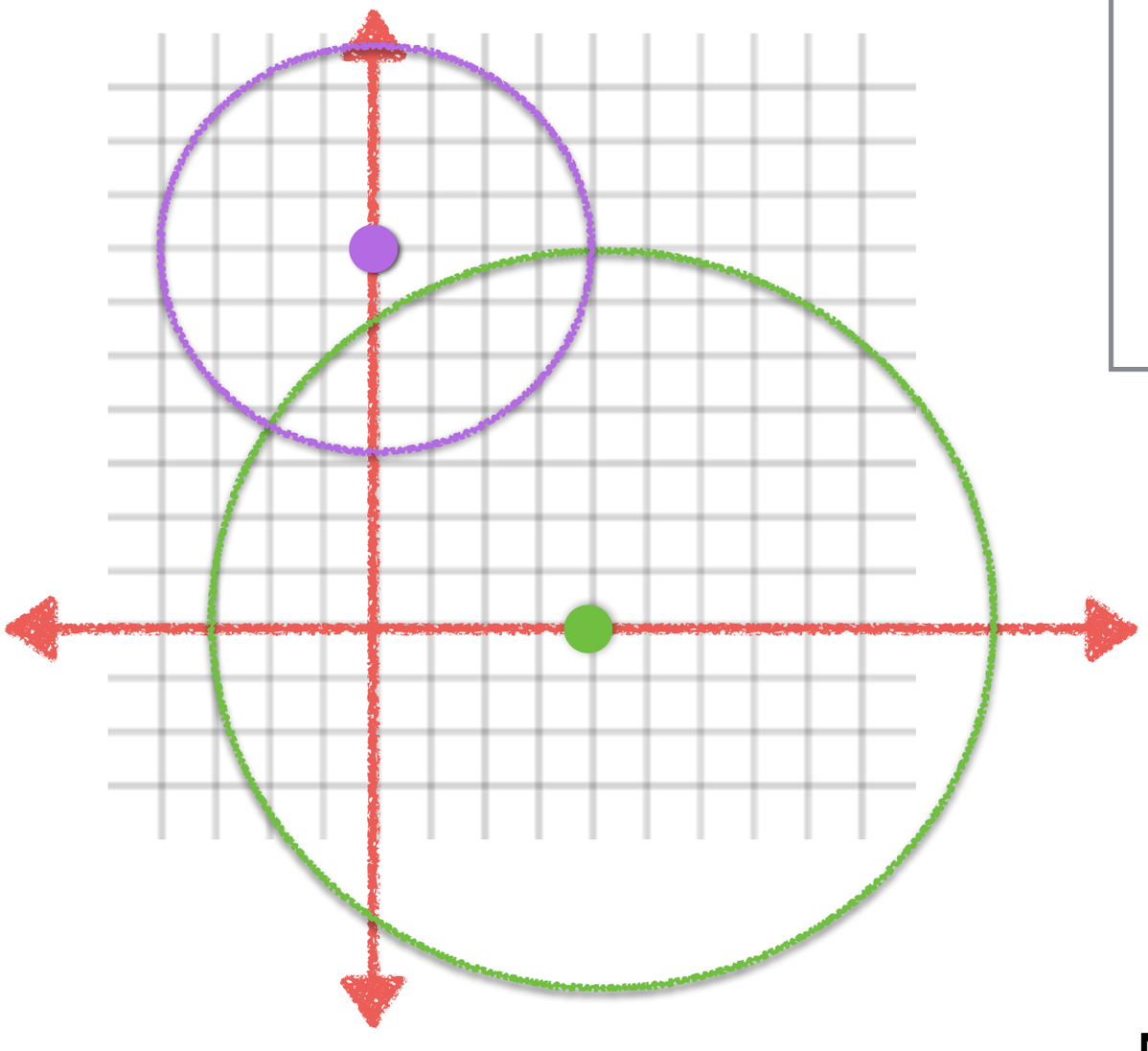
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(x_2, y_2) = (4,0)$

$(x_1, y_1) = (x, y)$

$d = 7$

Localization



System of equations

$$(x - 4)^2 + y^2 = 7^2$$

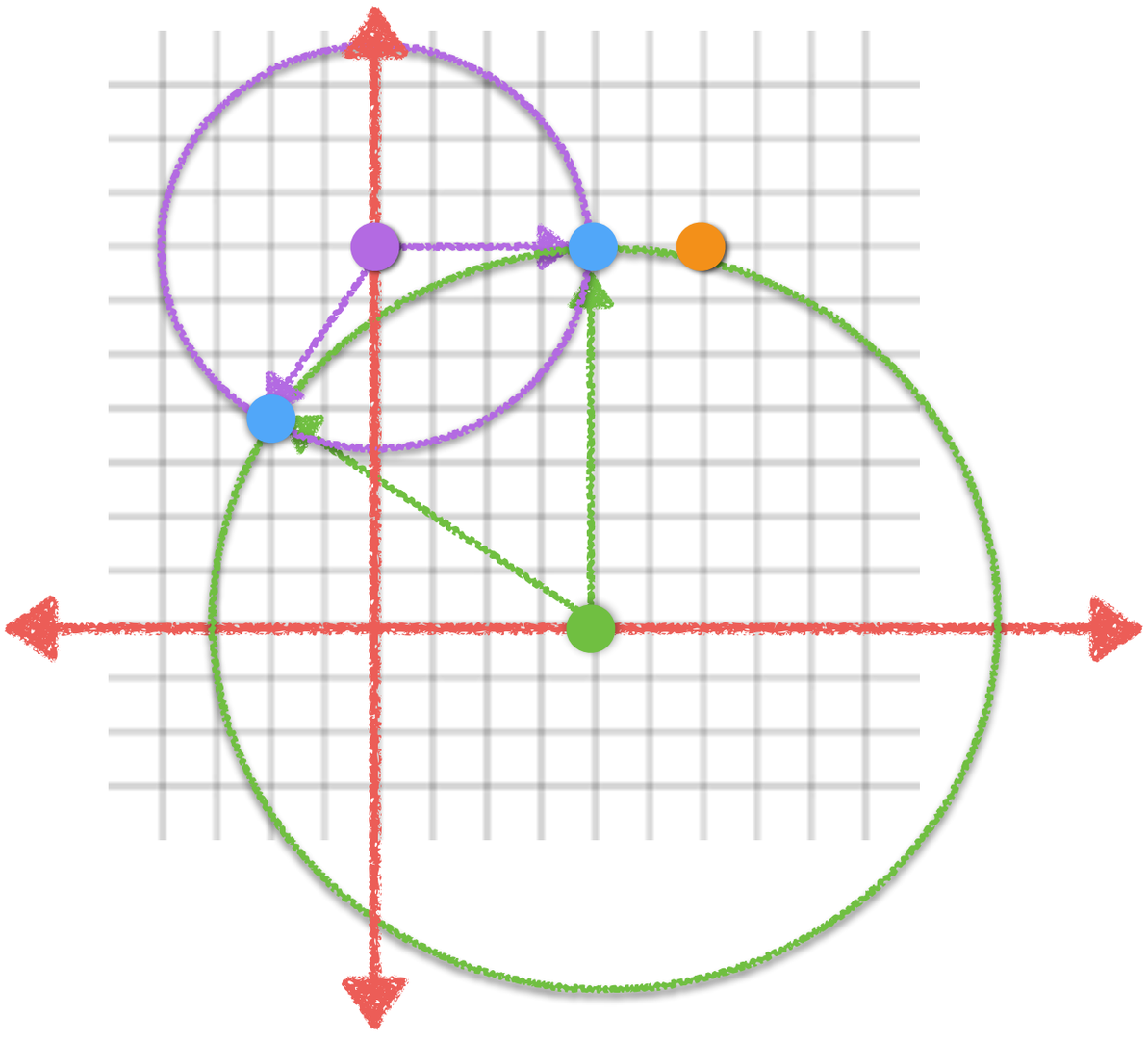
$$x^2 + (y - 7)^2 = 4^2$$

2 possibilities

$$(4, 7)\ ,\left(-\frac{132}{65}, \frac{231}{65}\right)$$

Not enough info to solve :(

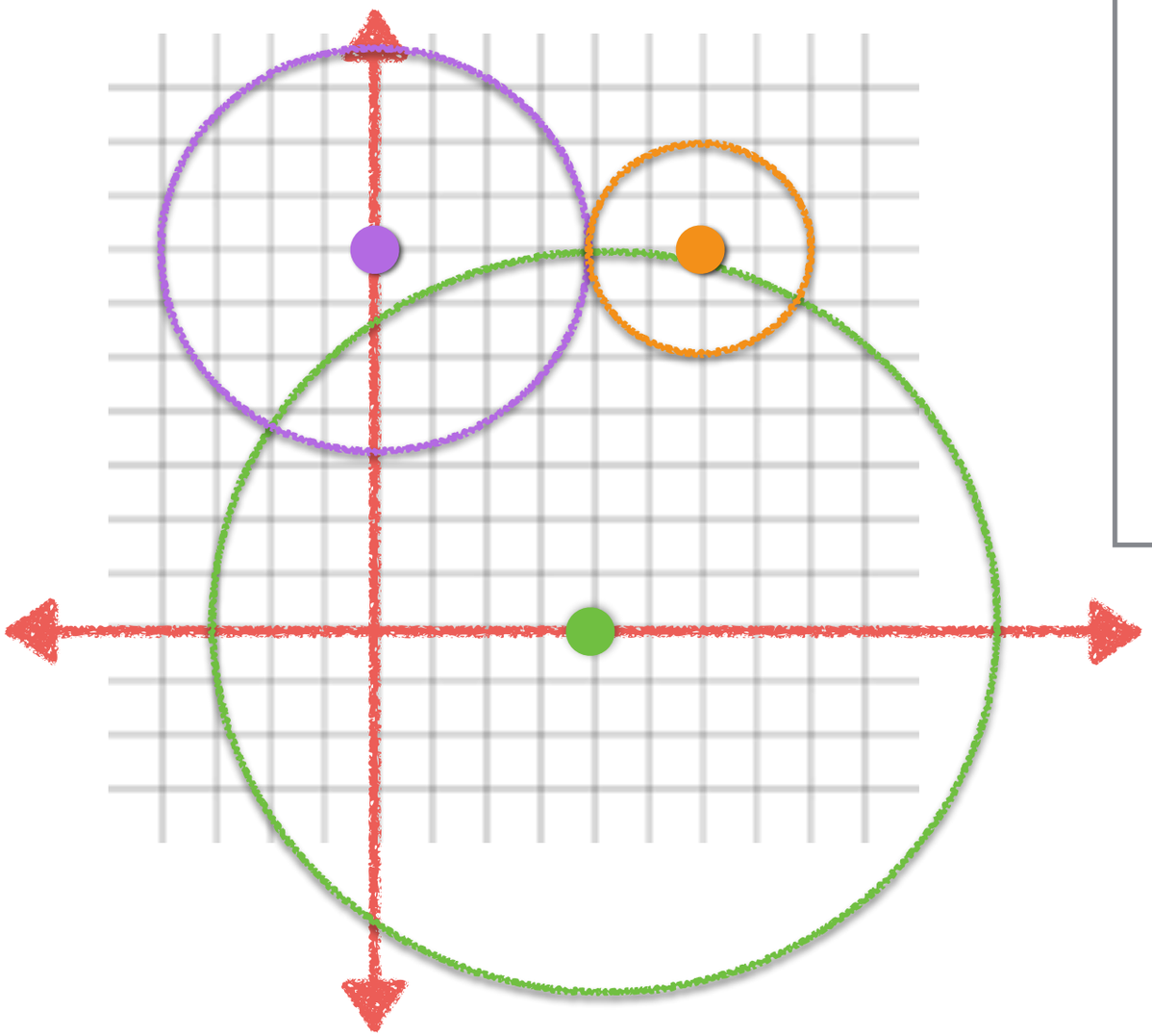
Localization



2 possibilities

Now, I can tell I am 2 units from (6,7)

Localization



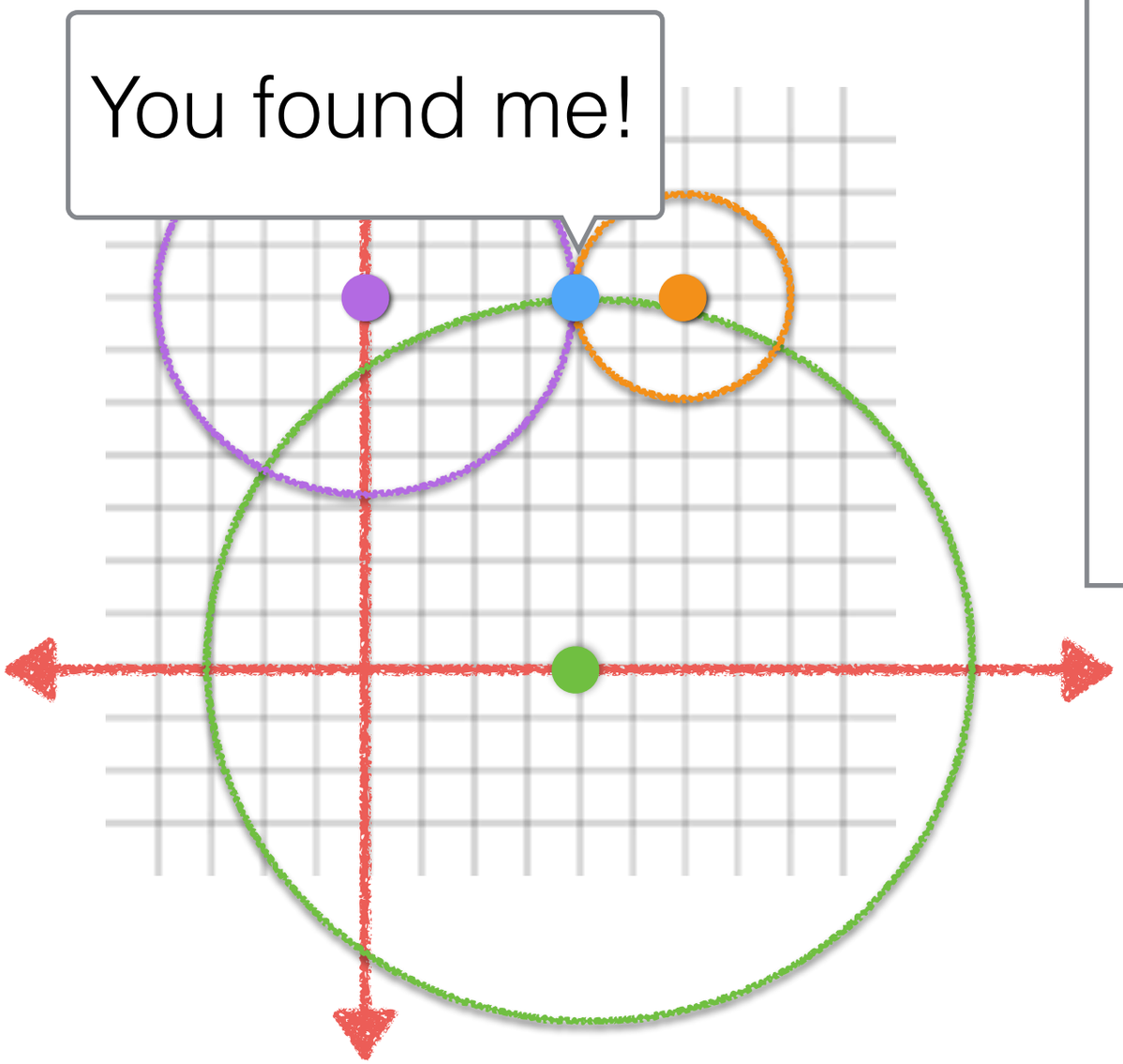
System of equations

$$(x - 4)^2 + y^2 = 7^2$$

$$x^2 + (y - 7)^2 = 4^2$$

$$(x - 6)^2 + (y - 7)^2 = 2^2$$

Localization



System of equations

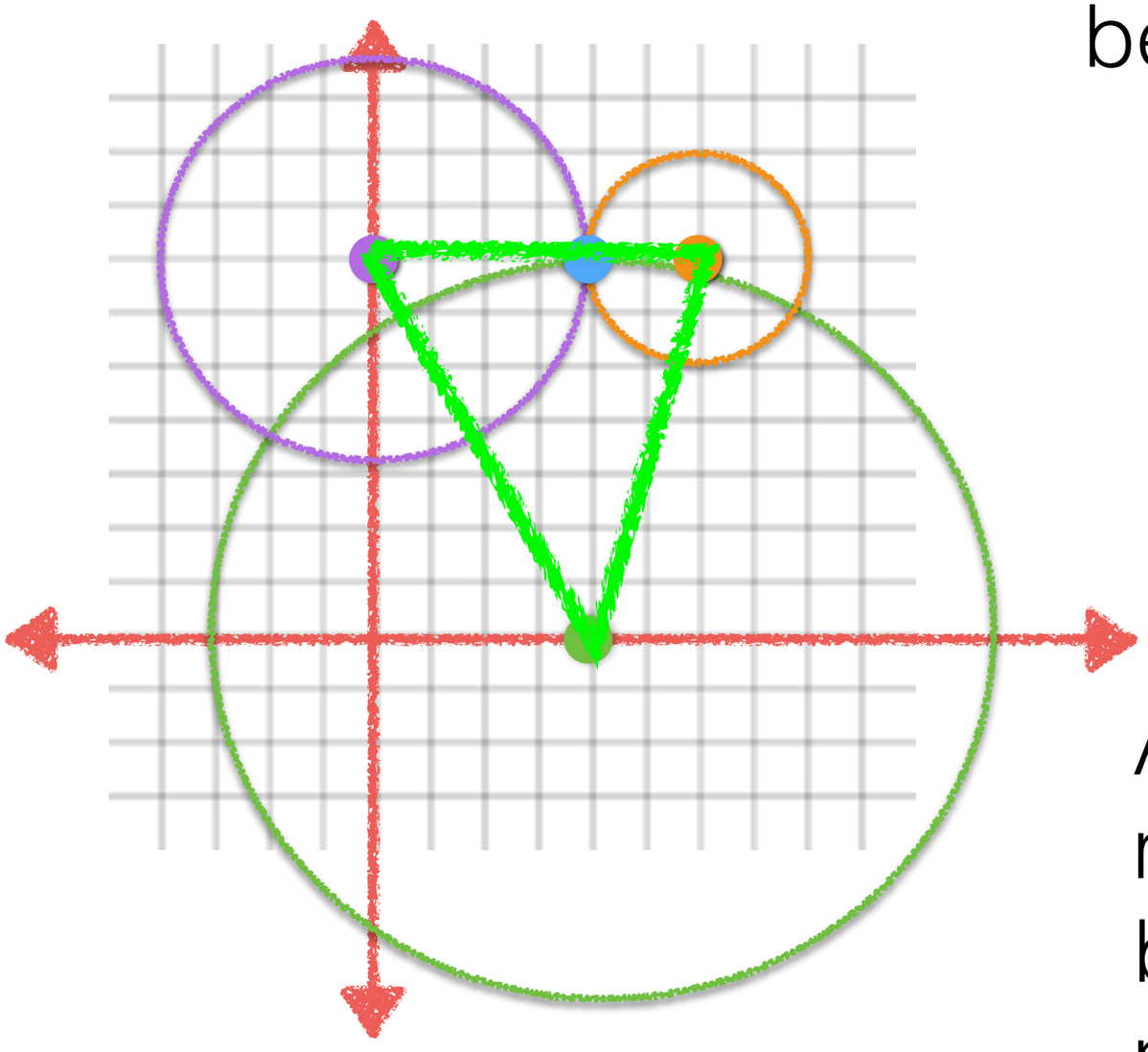
$$(x - 4)^2 + y^2 = 7^2$$

$$x^2 + (y - 7)^2 = 4^2$$

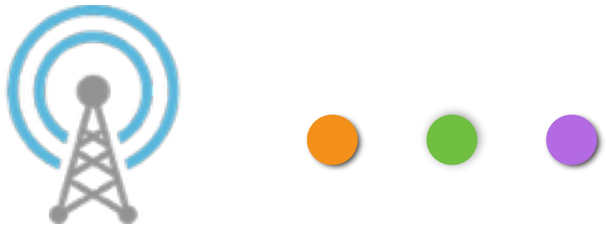
$$(x - 6)^2 + (y - 7)^2 = 2^2$$

Just one solution!
$(4, 7)$

Cell Tower Triangulation

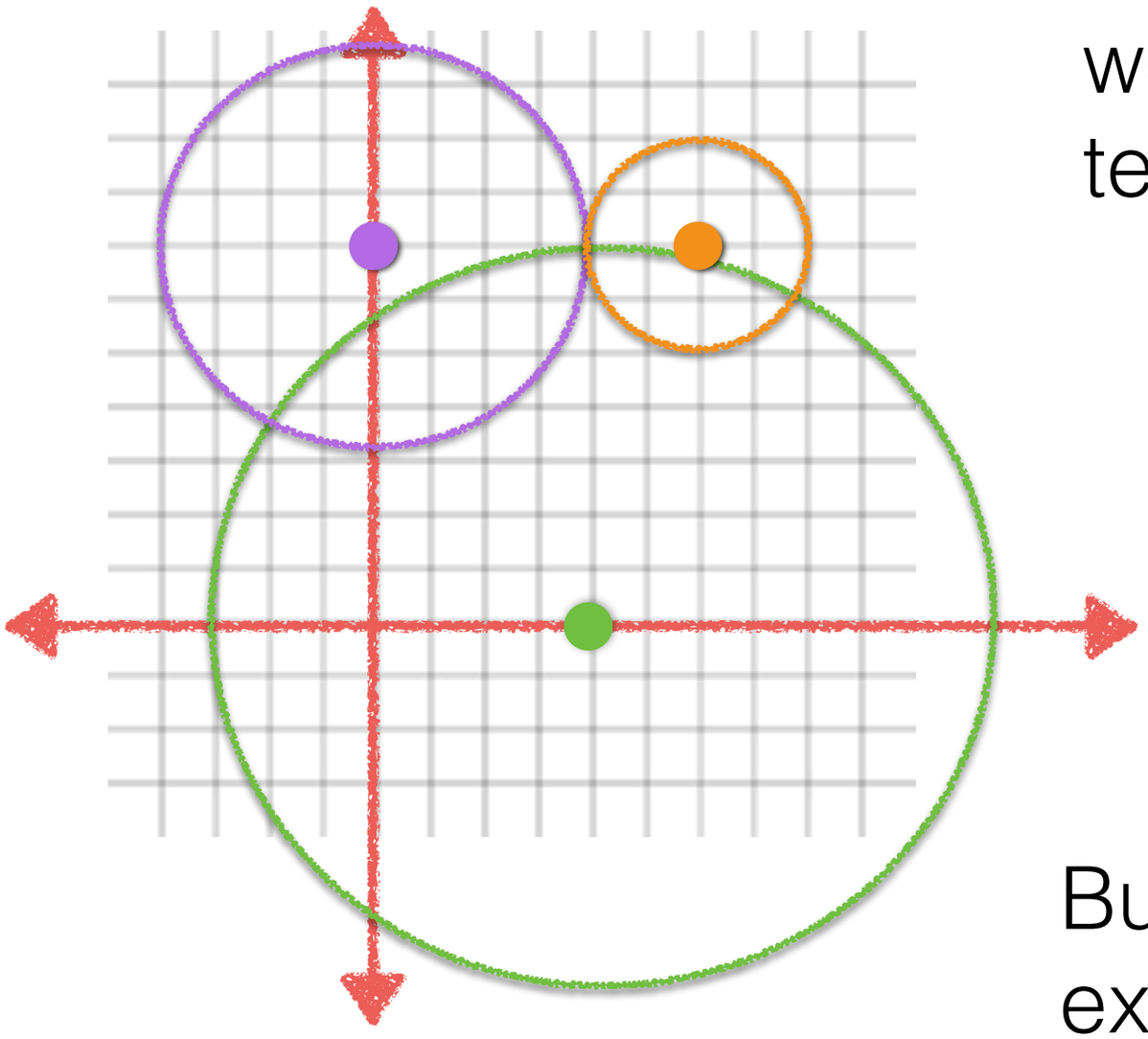


Each of these landmarks can be cell phone towers.



A tower's location can be marked exactly,
but how can cell phones measure distance?

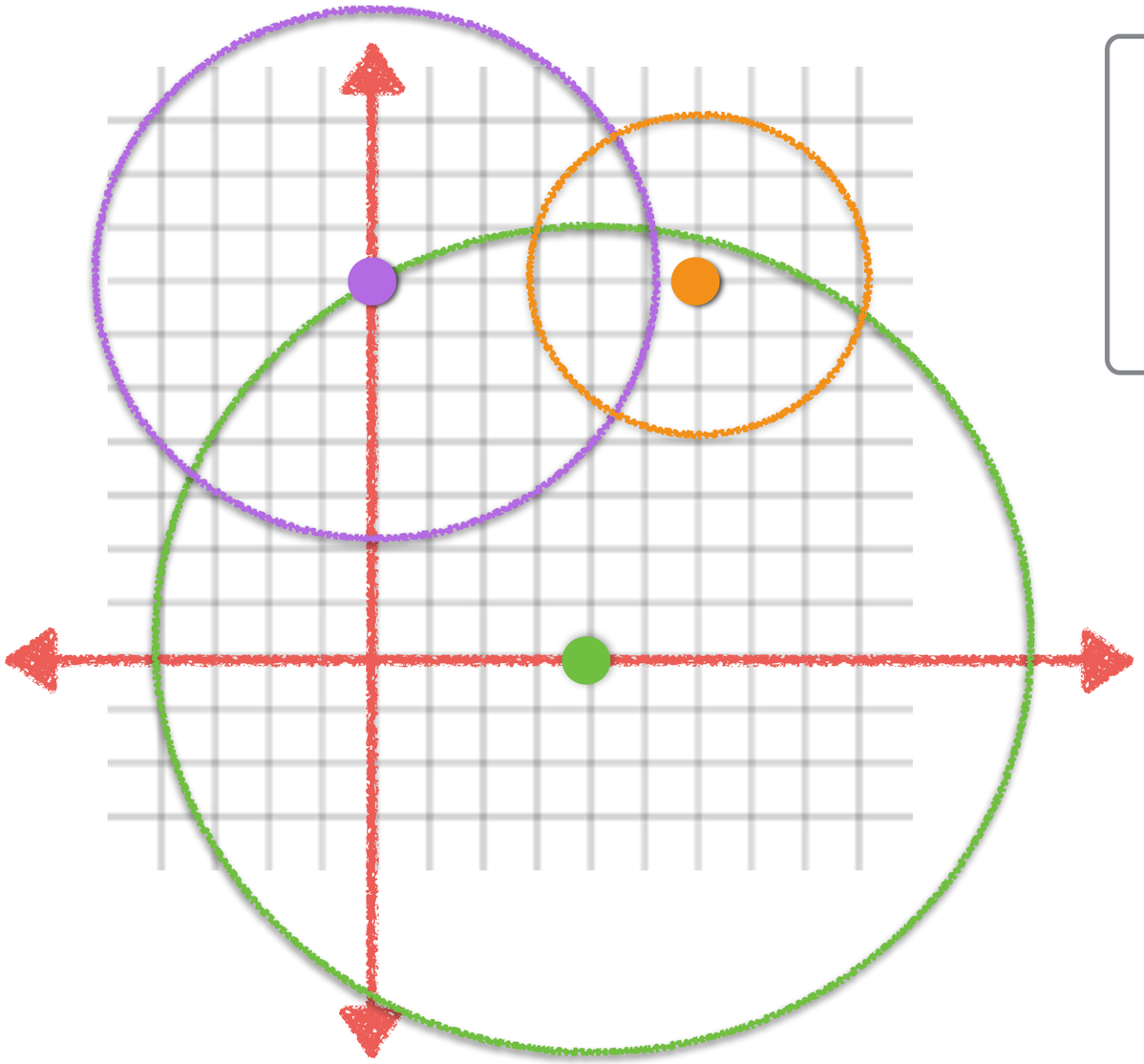
Localization



If we know exactly how far we are from 3 points, we can tell exactly where we are.

But what if we don’t know exactly how far away we are?

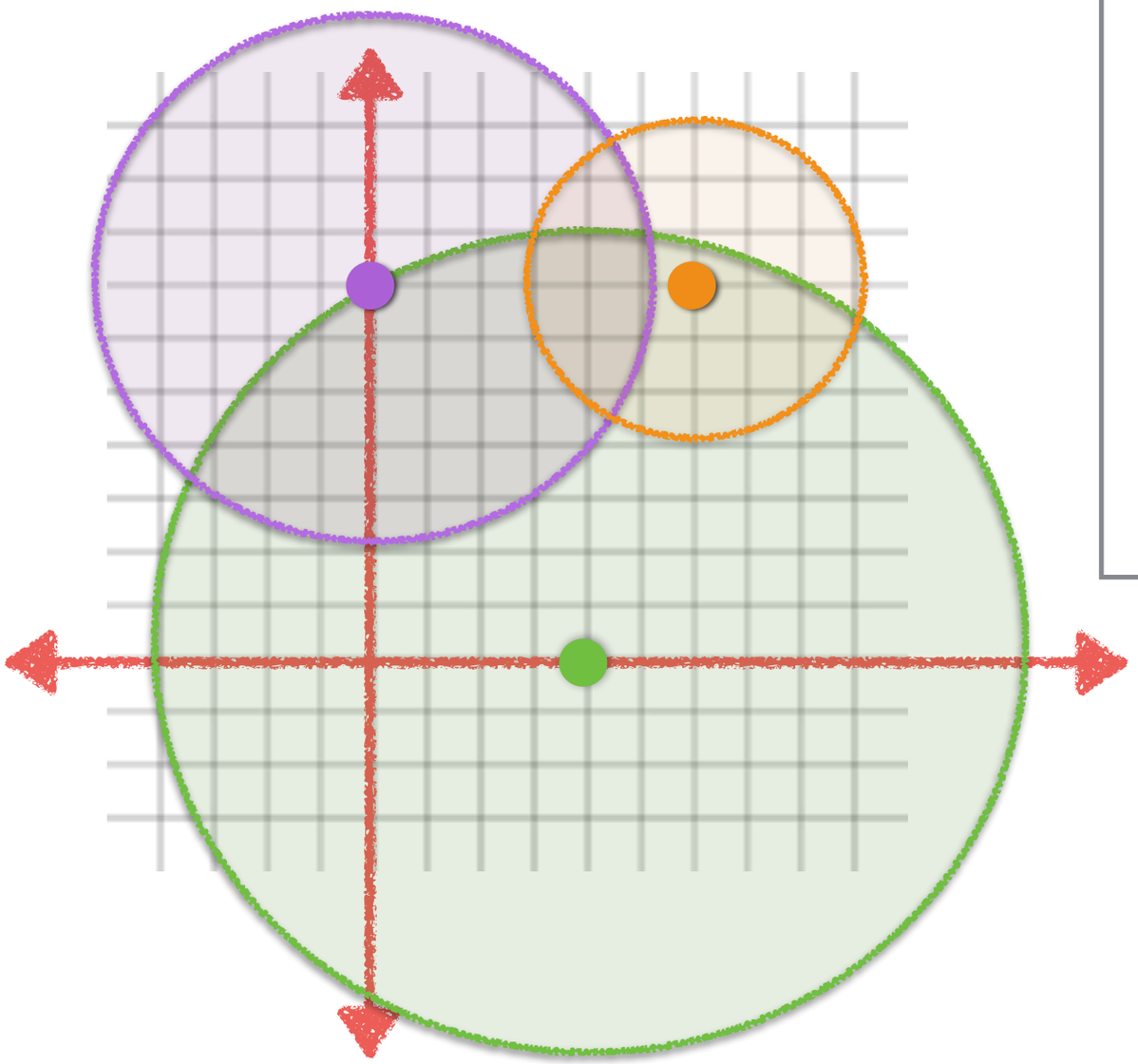
Localization



hello!
I'm lost :(
can you find me?

I know I am
- ≤ 5 units from (0,7)
- ≤ 8 units from (4,0)
- ≤ 3 units from (6,7)

Localization



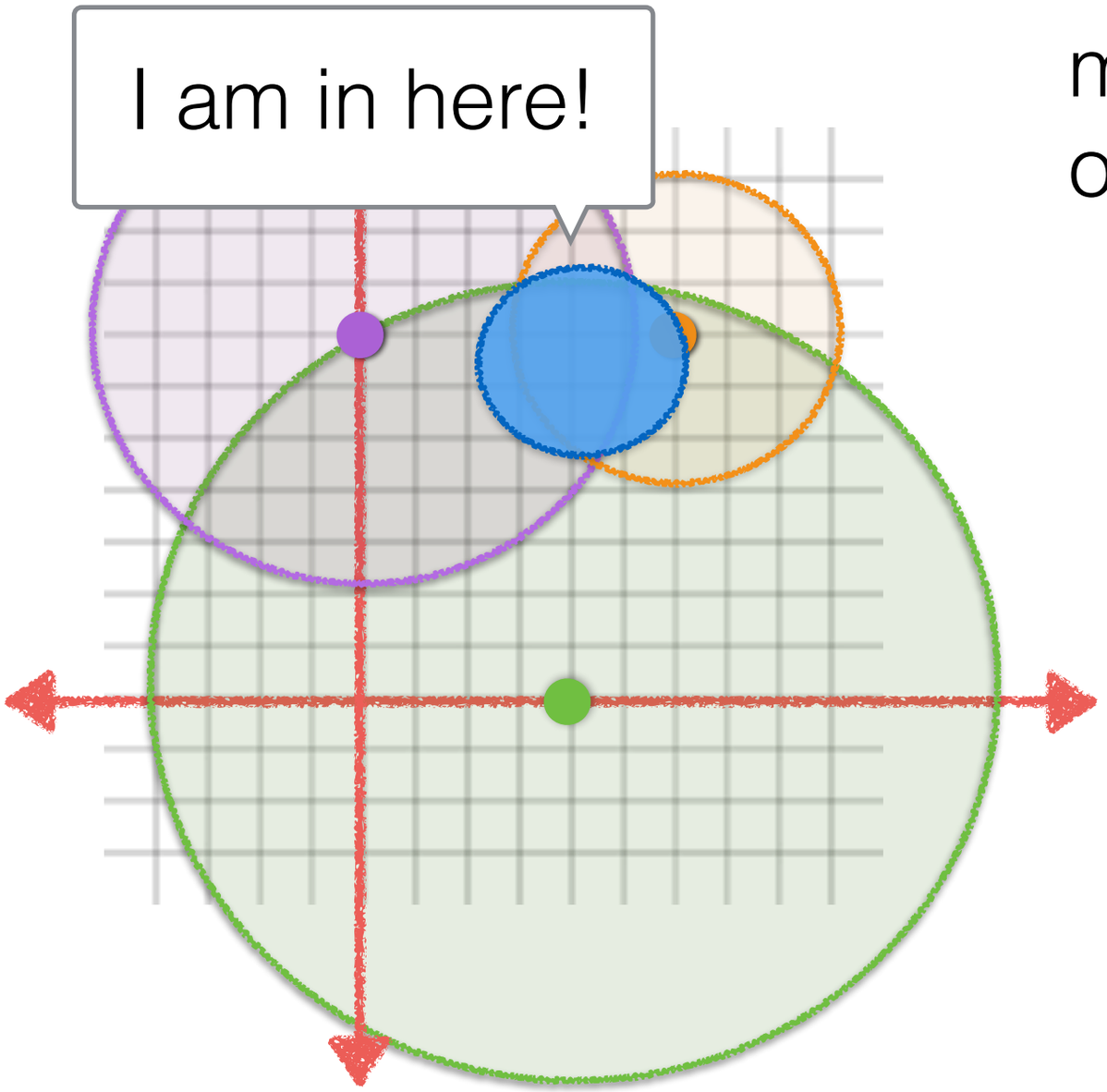
System of inequalities

$$(x - 4)^2 + y^2 \leq 7^2$$

$$x^2 + (y - 7)^2 \leq 4^2$$

$$(x - 6)^2 + (y - 7)^2 \leq 2^2$$

Localization

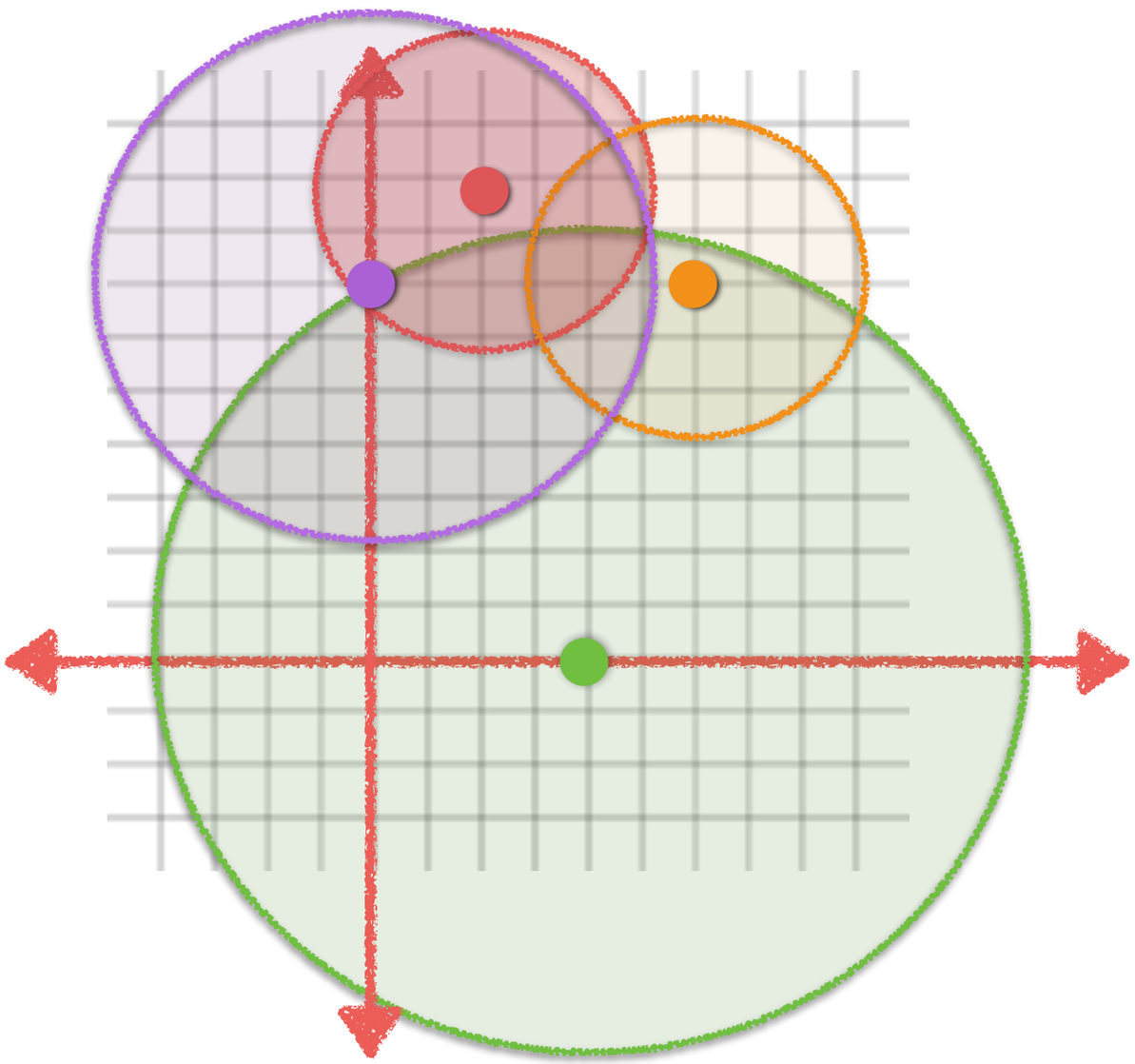


We can use the points to make a new circle where our friend should be within.

Given 3 points, you can construct a circle (interesting challenge).

How can we make our approximation better?

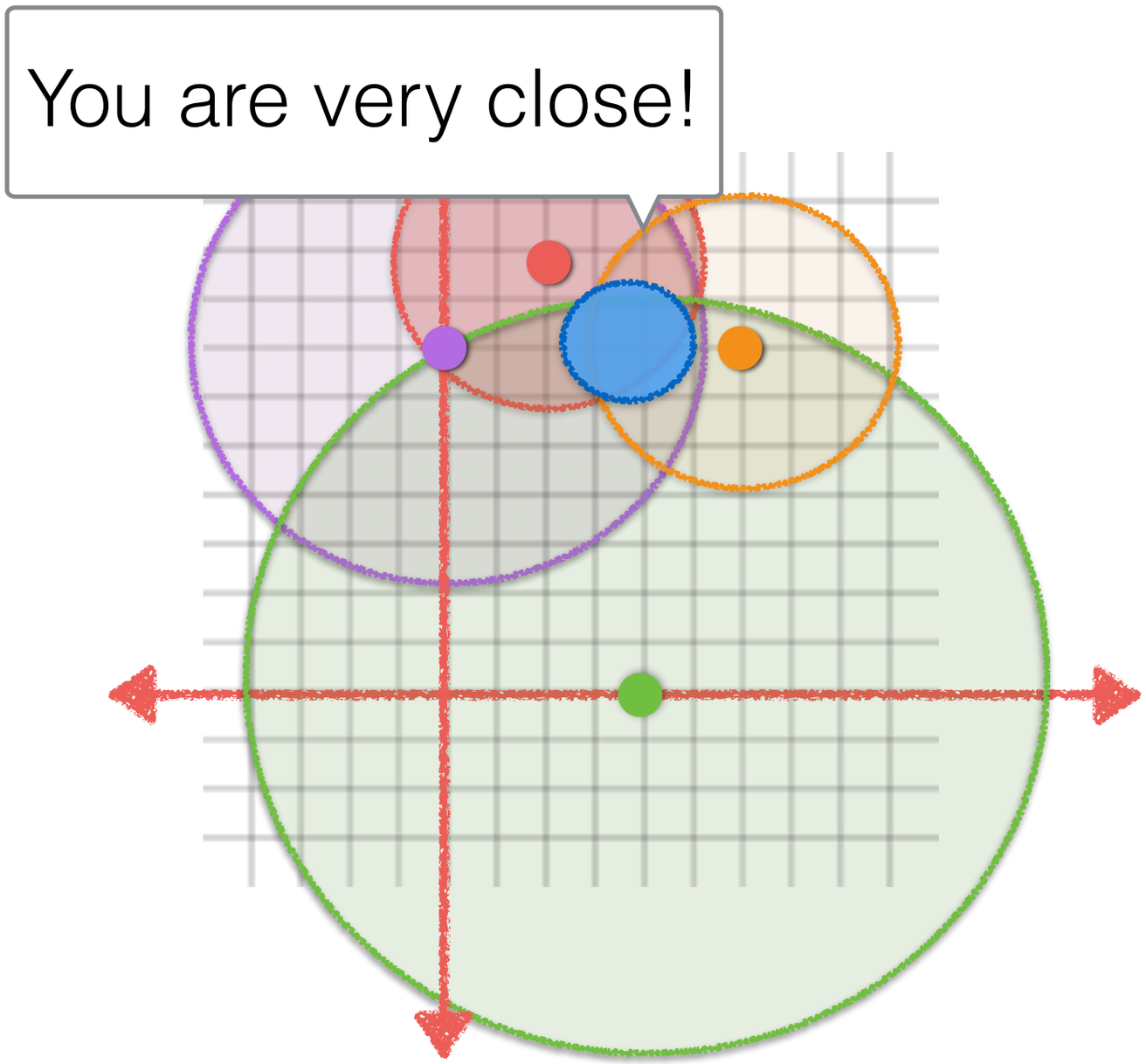
Localization



How can we make our approximation better?

Add more measurements would help!

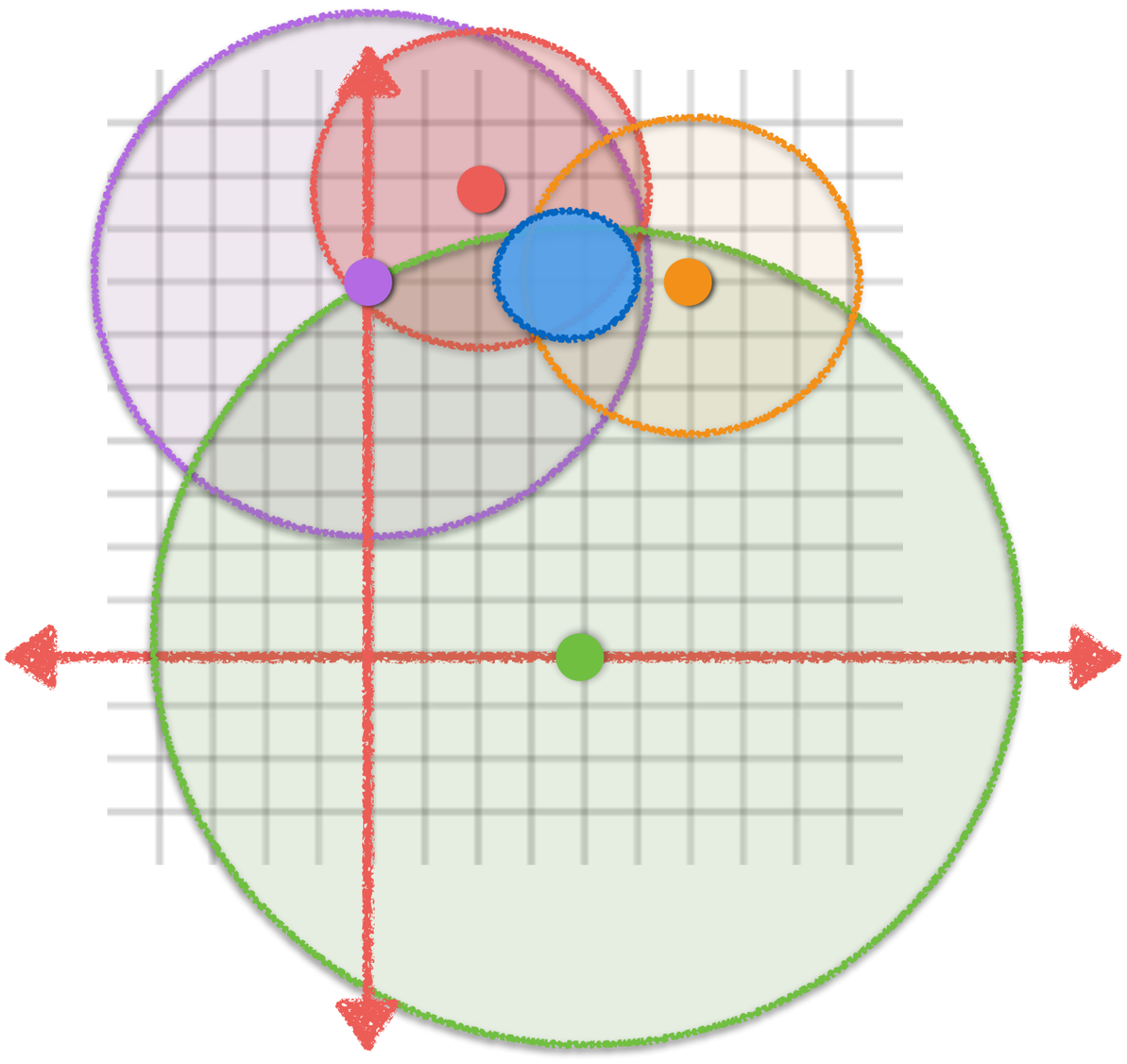
Localization



How can we make our approximation better?

Add more measurements would help!

Cell Phone Tracking without GPS



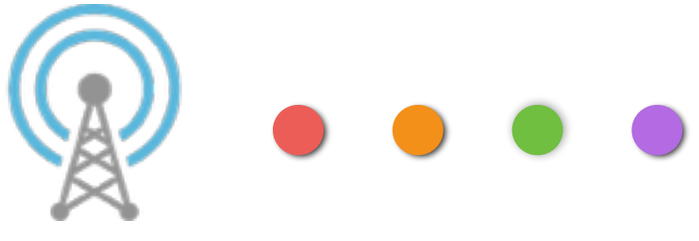
Recap

So what did we need to know in order to find our friend?

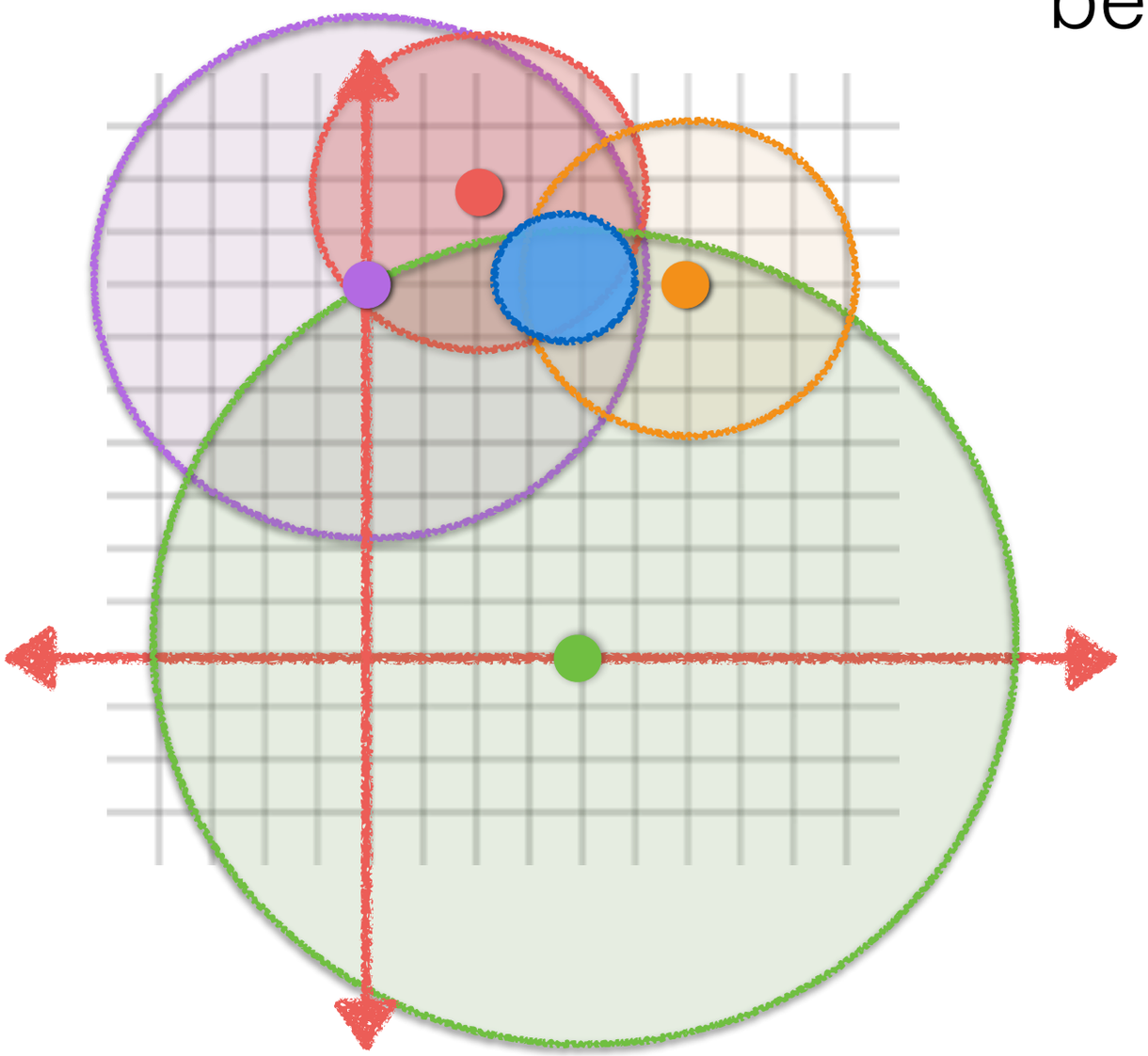
- the exact position
- approximate distances

from 3 or more landmarks.

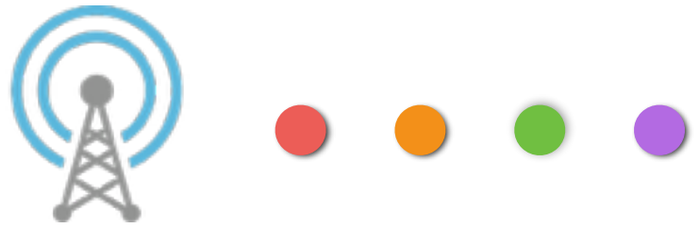
Each of these landmarks can be cell phone towers.



Cell Phone Tracking without GPS

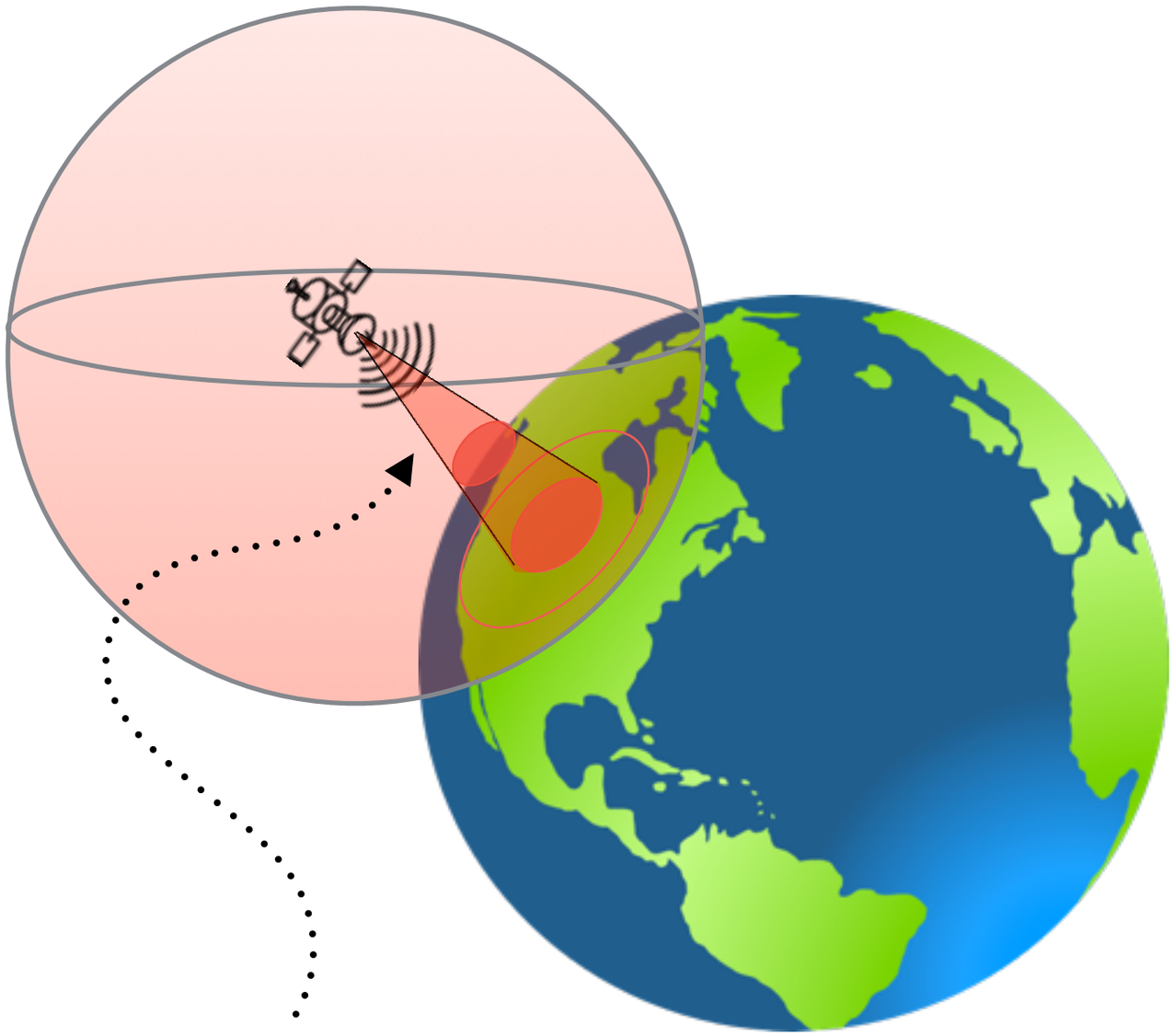


Each of these landmarks can be cell phone towers.



GPS

GPS basically does the same thing,
but in 3D, with spheres!



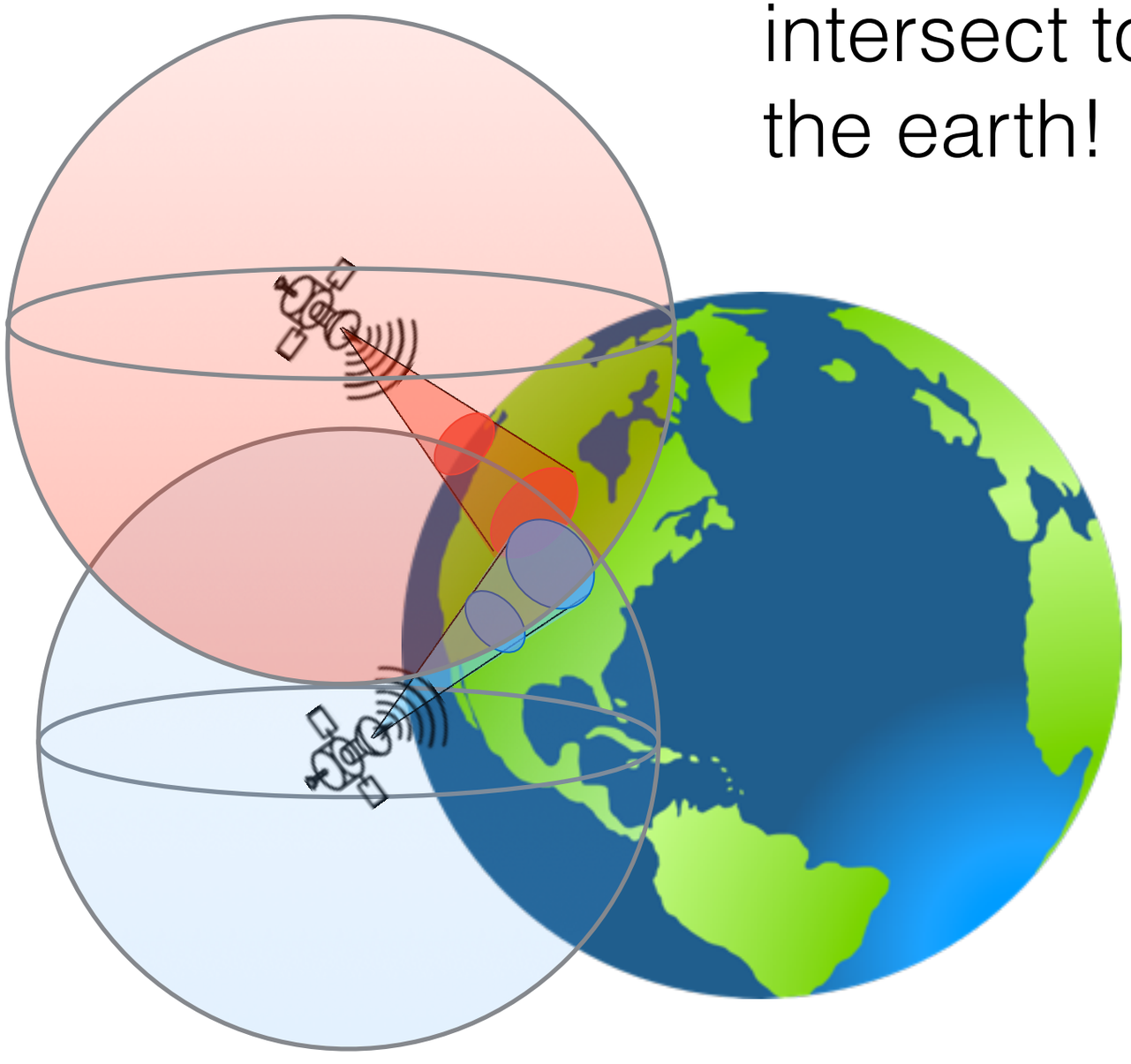
Your GPS device calculates this distance

The satellite tells your GPS device exactly where it is and your GPS device is constantly calculating the distance to the satellite!

Using the distance and position, a circle can be drawn where the satellite cone and the earth sphere intersect.

GPS

With enough circles, your GPS device calculates where they intersect to tell where you are on the earth!

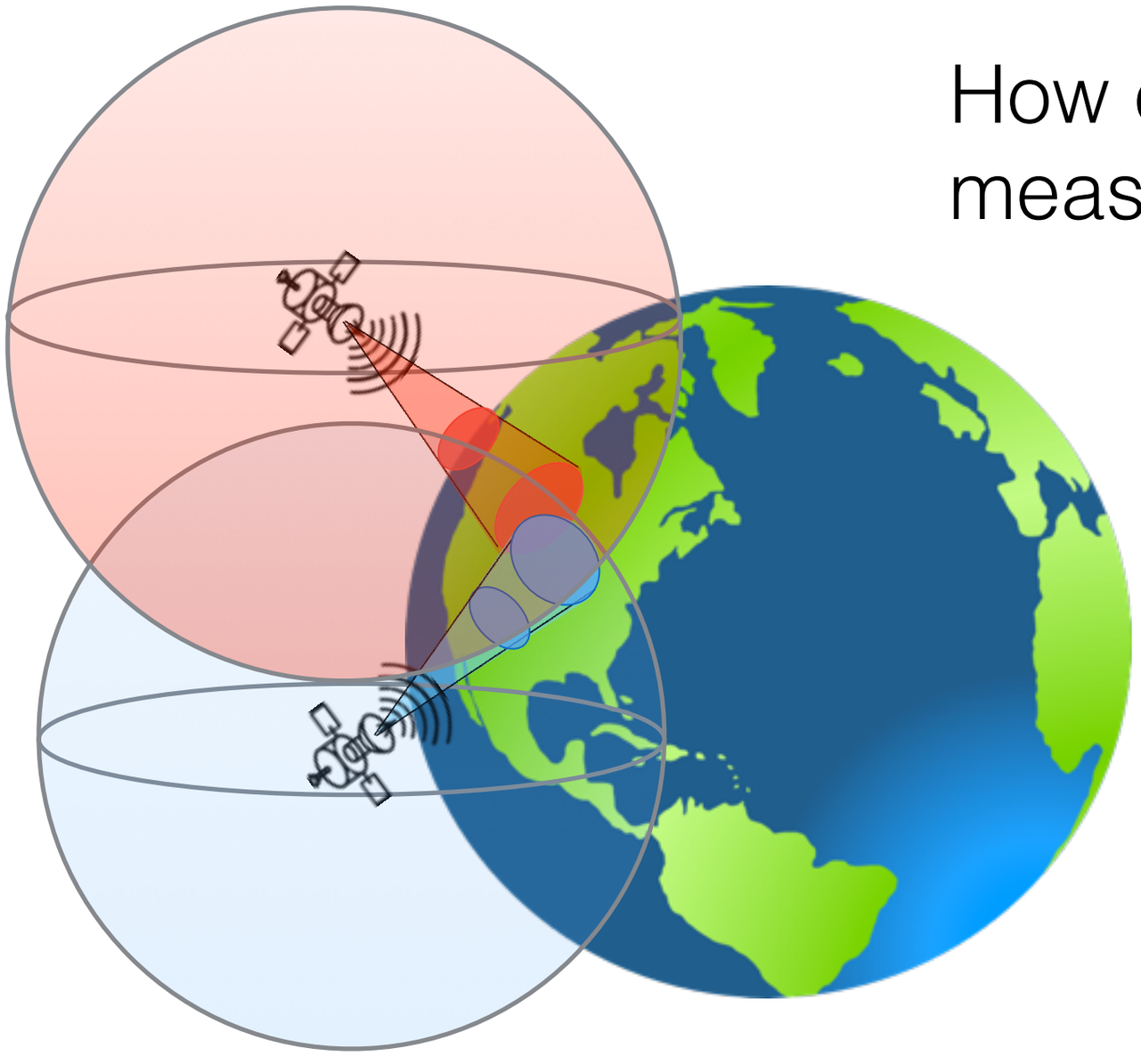


The location is given in coordinates of latitude and longitude.

Then, your map program looks up what street you are on based on your latitude and longitude.

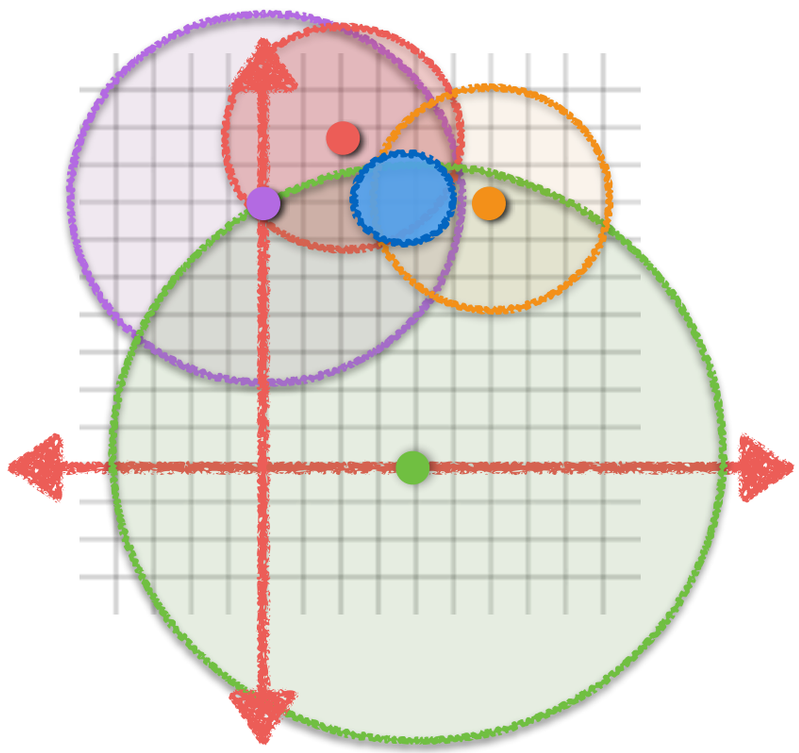
GPS

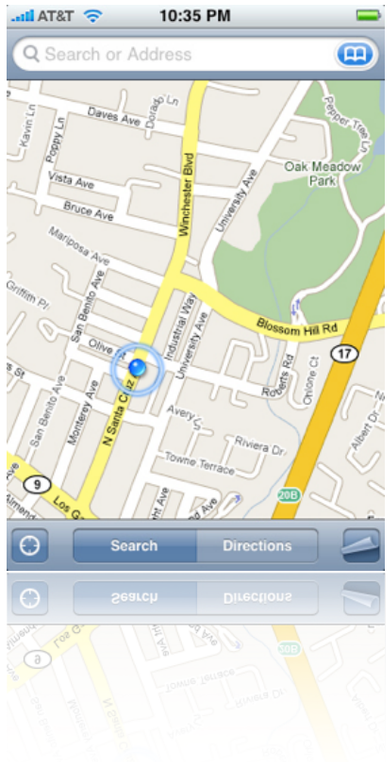
A GPS satellite sends the position and the time to a GPS device.



How does your GPS device measure distance to the satellite?

GPS





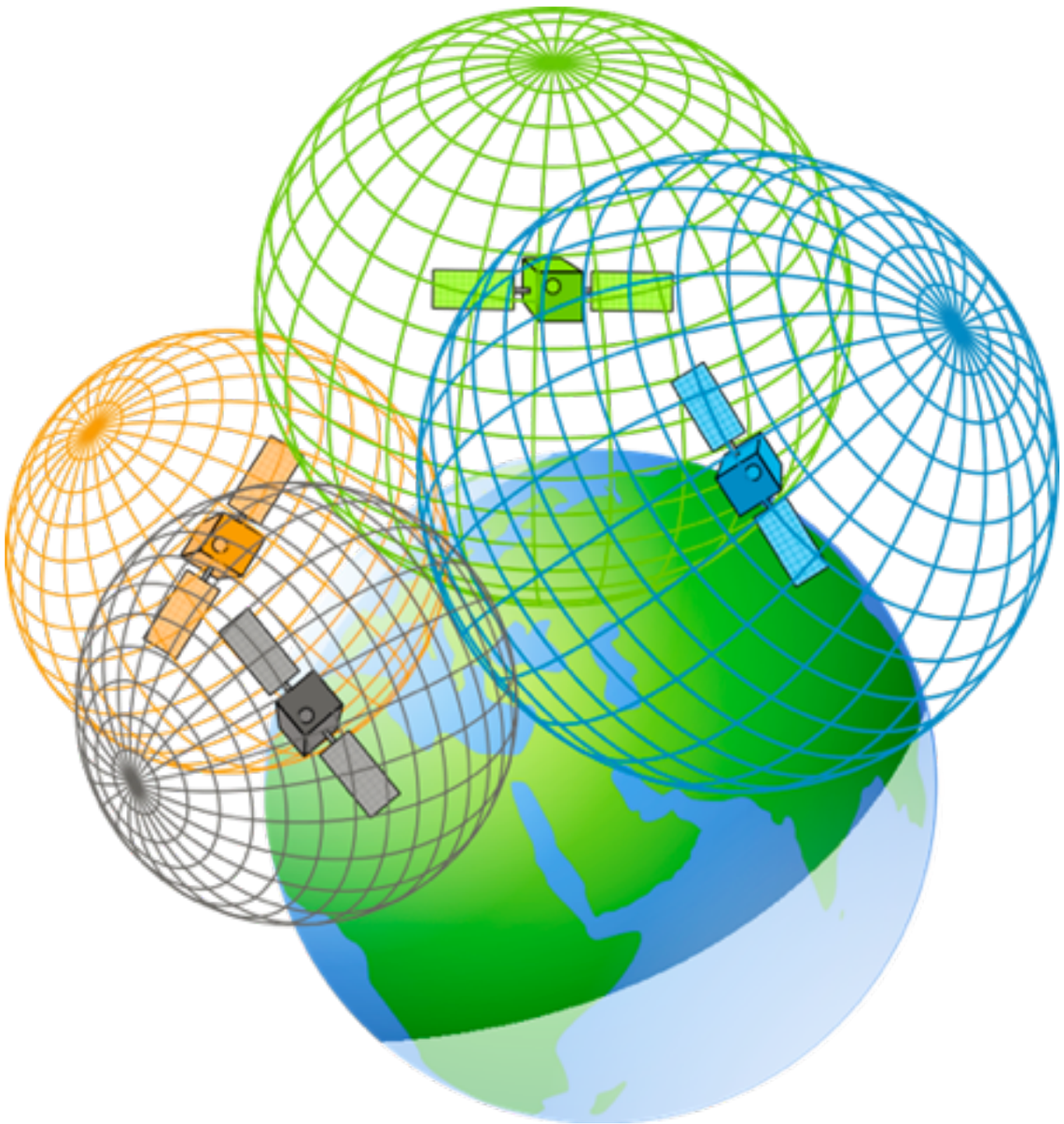
GPS has a network of 32 satellites.

A GPS satellite broadcasts its location and time. It has a very precise clock called an atomic clock.

GPS devices calculate how far away they are from a satellite using the satellite's time and location signal.

If your GPS can receive signals from 4 or more satellites then you can tell your position very accurately!

GPS



The next time you see your GPS location on your phone or in your car, you know your phone is listening to 4 satellites in space (!) and using geometry to calculate exactly where you are!

Thanks for listening!