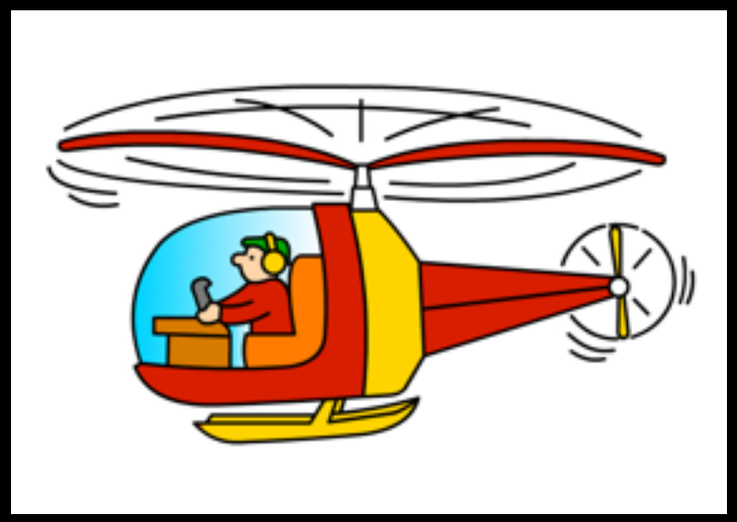
Nicolas Bauer

Math Busted: Helicopters Taking off without Moving Rotors!



How is this video possible?

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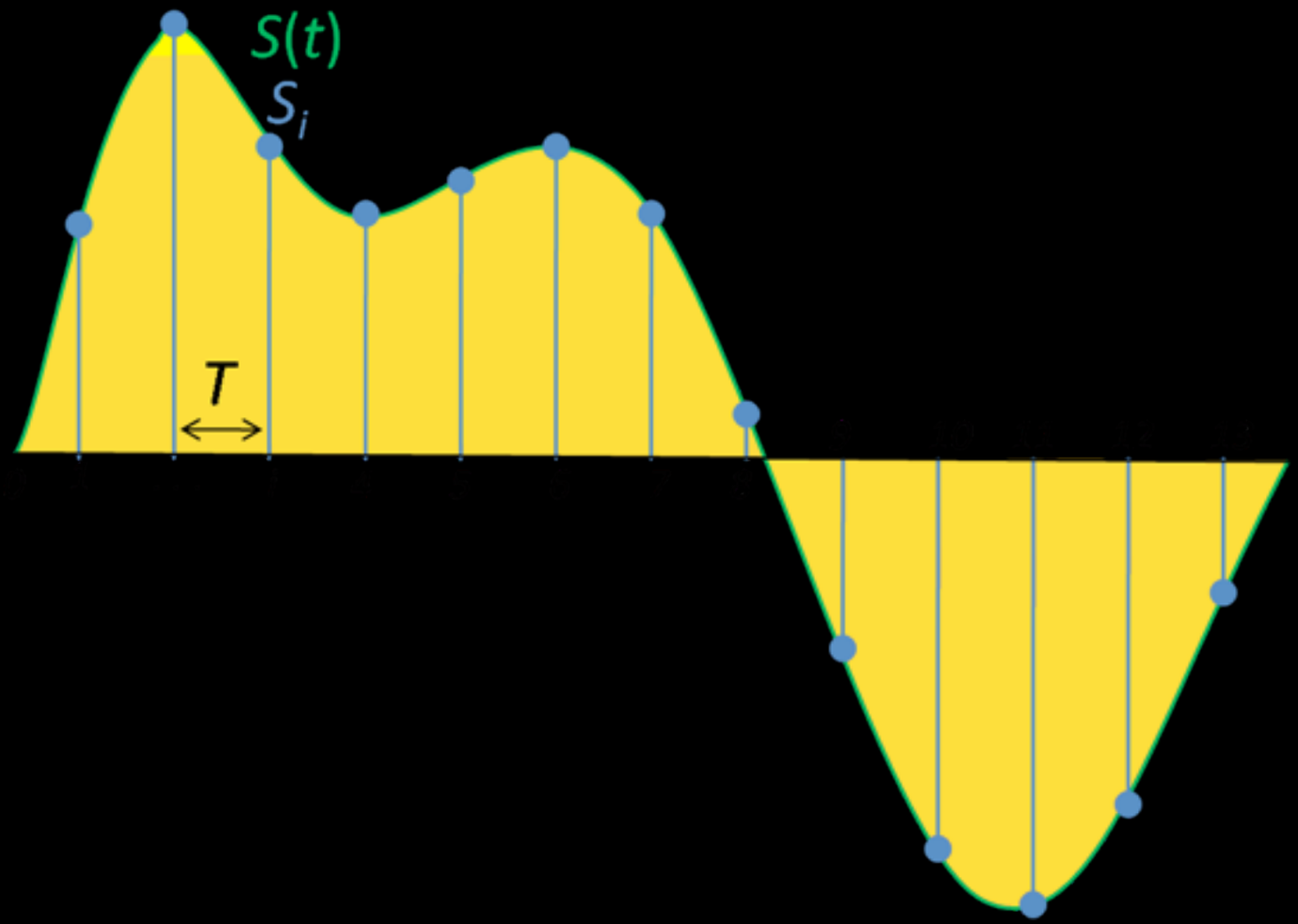
Outline

- cameras and how they work
- sampling, what is it?
- rotation as a function
- sampling things that rotate
- aliasing, what is it?
- how aliasing explains interesting observations

Let’s talk about sampling



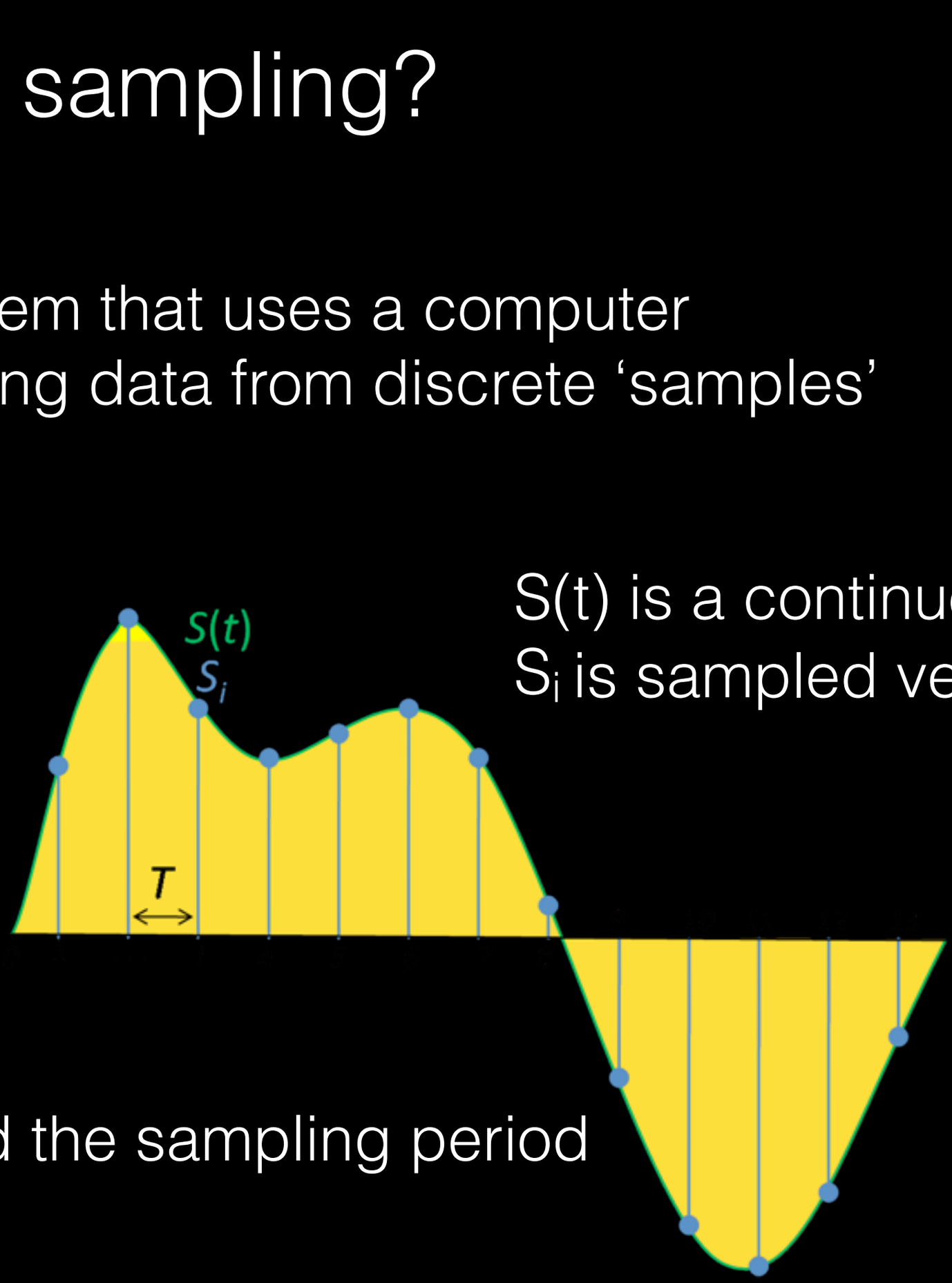
Cameras sample

Cameras are digital devices and record a series of images at a certain rate known as the ‘frame rate’ or ‘sample rate’

What is sampling?

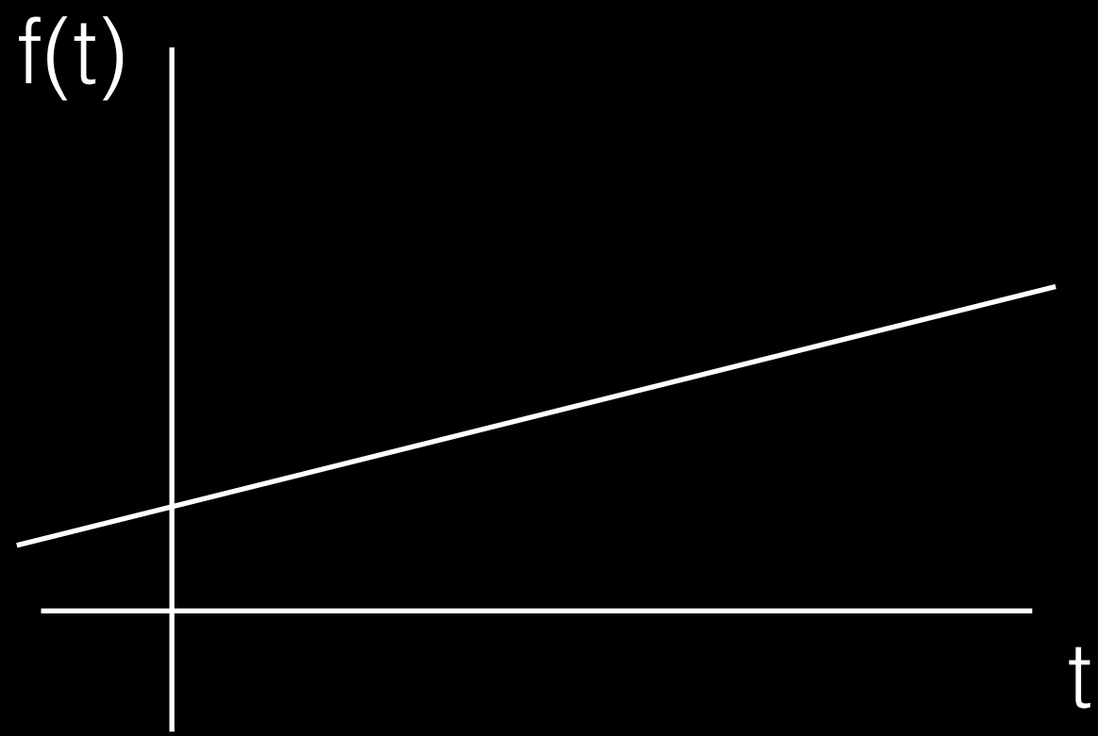
Every system that uses a computer
is computing data from discrete ‘samples’

S(t) is a continuous function
S_i is sampled version of S(t)

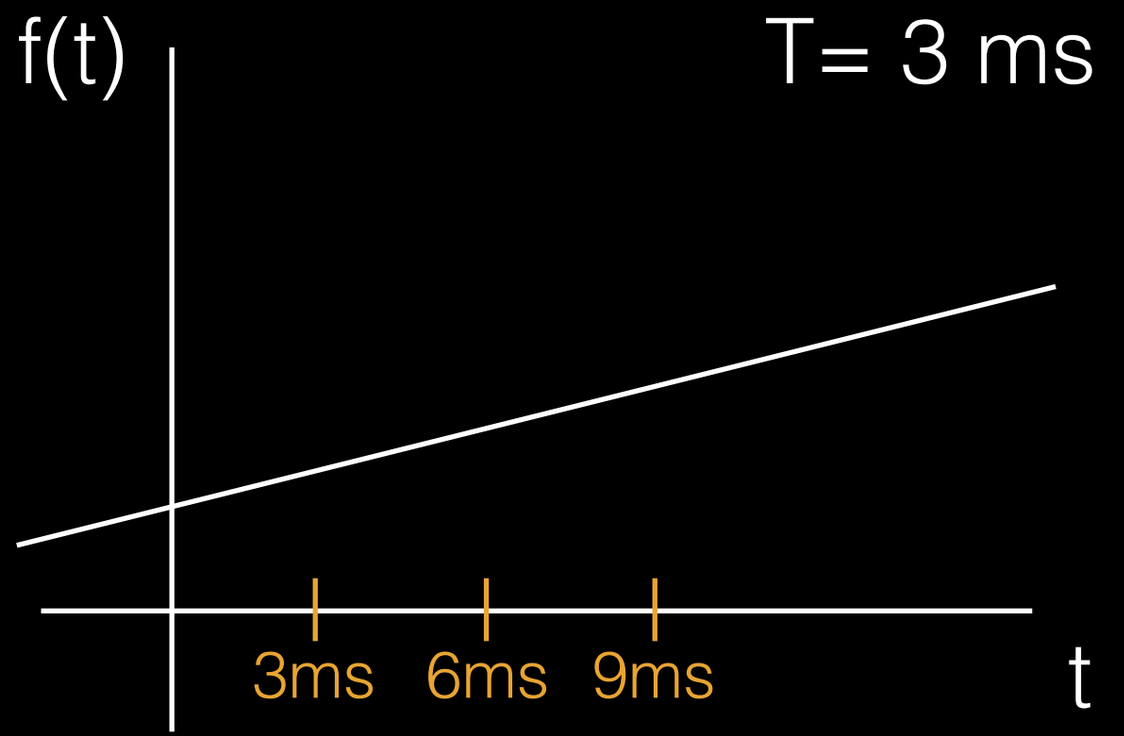


T is called the sampling period

$$f(t) = 0.5t+1$$



$f(t) = 0.5t + 1$



$f_k = 0.5kT+1$

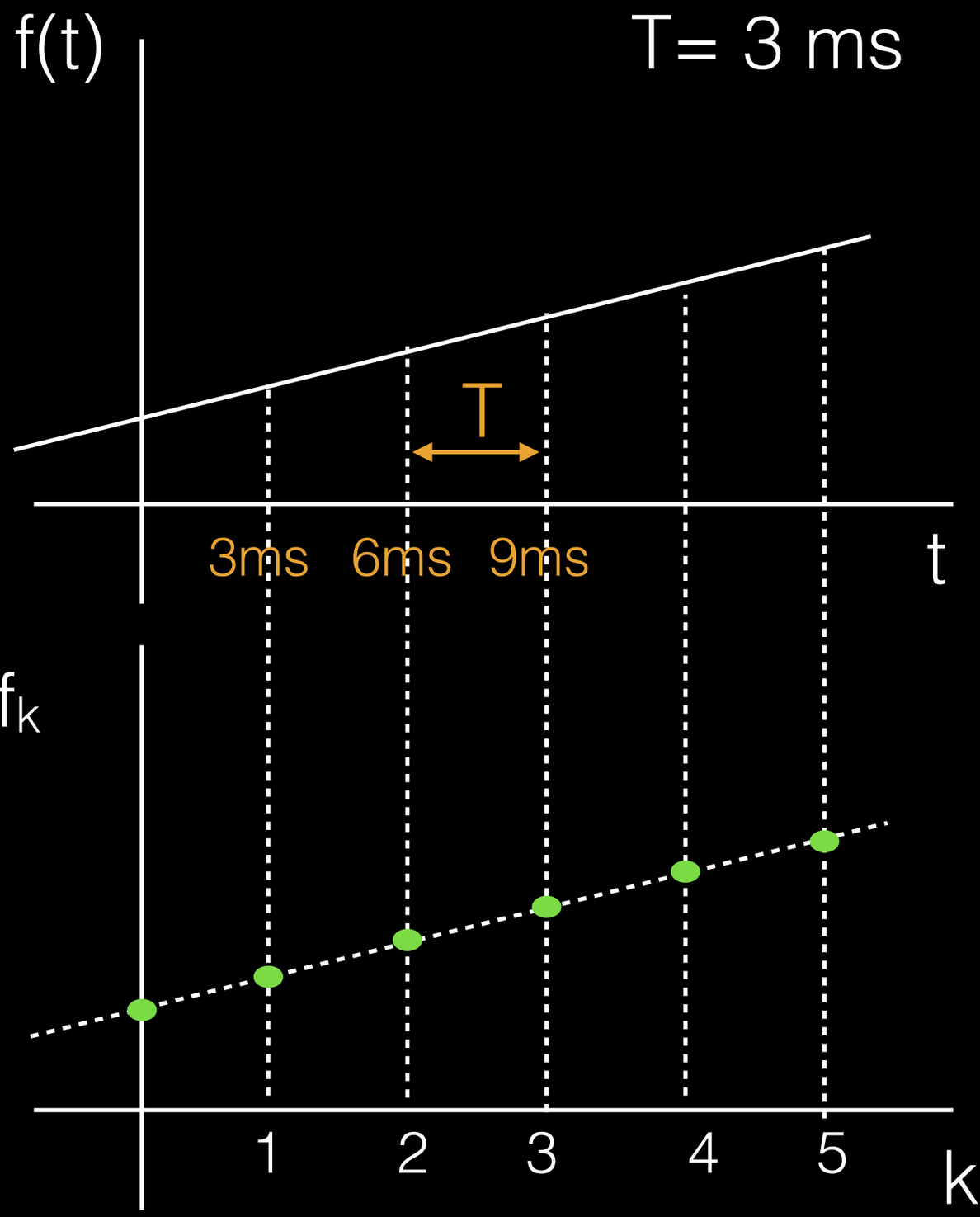
k = sample number
T = sampling interval

$f(t) = 0.5t+1$

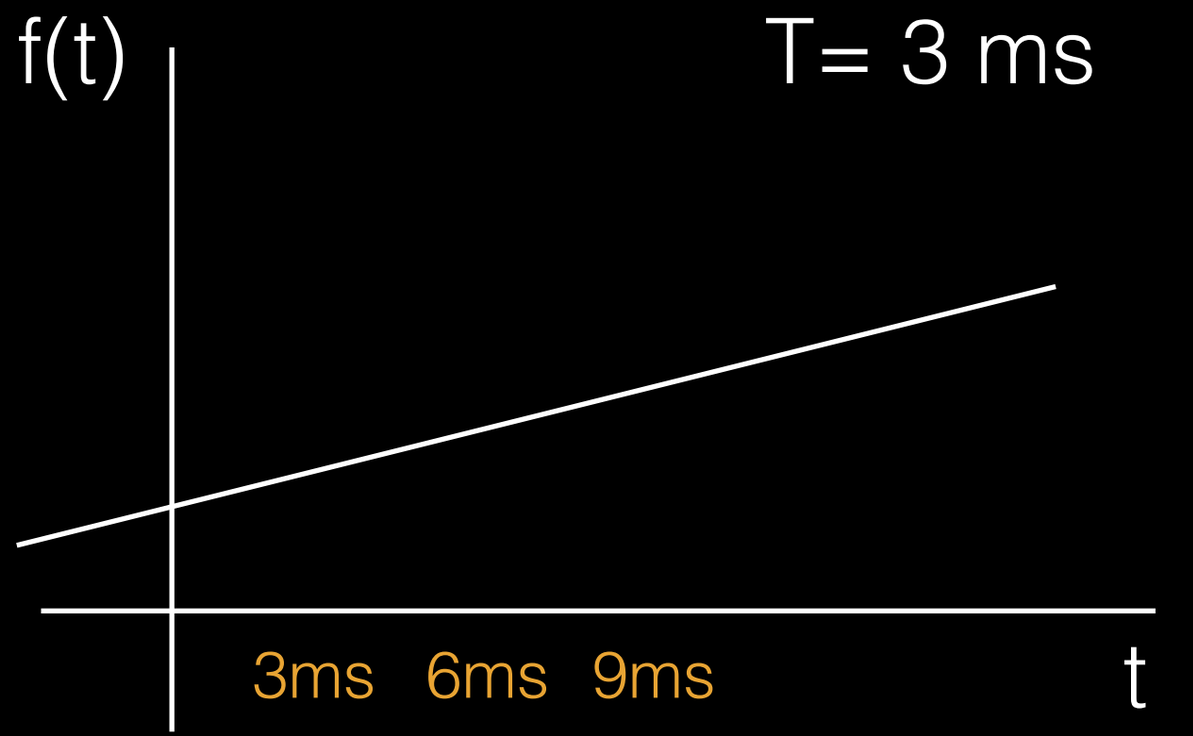
$f_k = 0.5kT+1$

k = sample number
T = sampling interval

k	kT
1	3ms
2	6ms
3	9ms



$f(t) = 0.5t+1$



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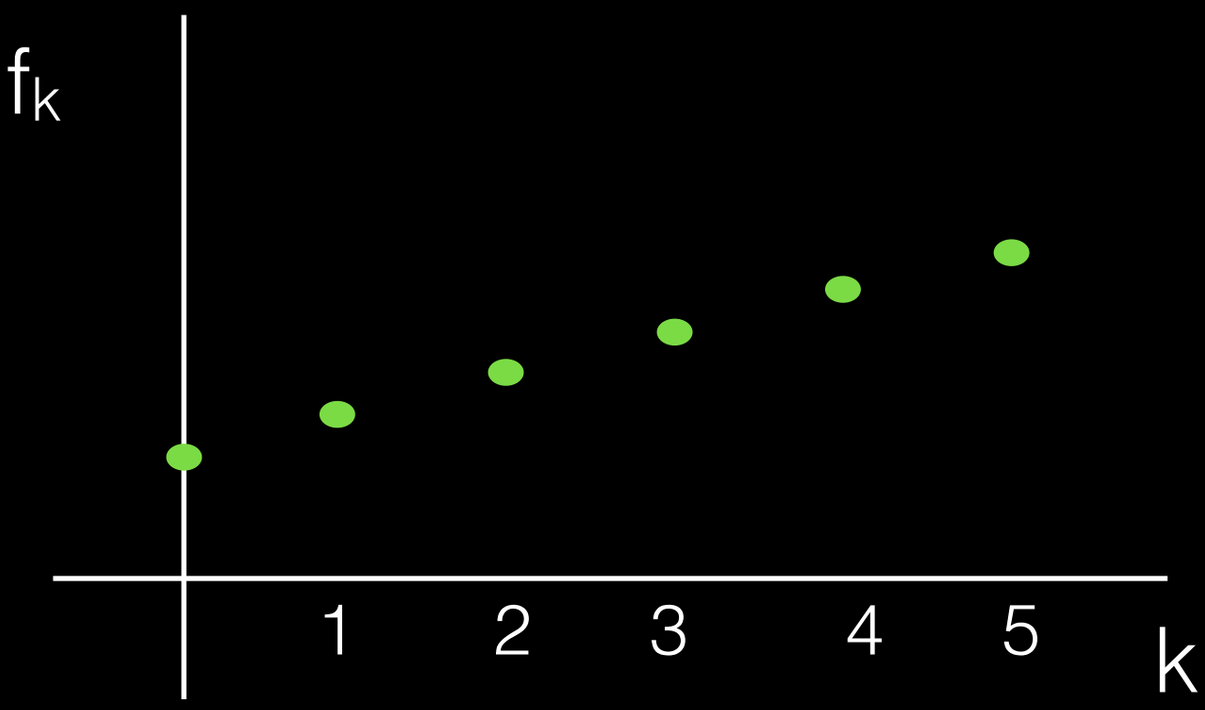


What kind of other things sample?

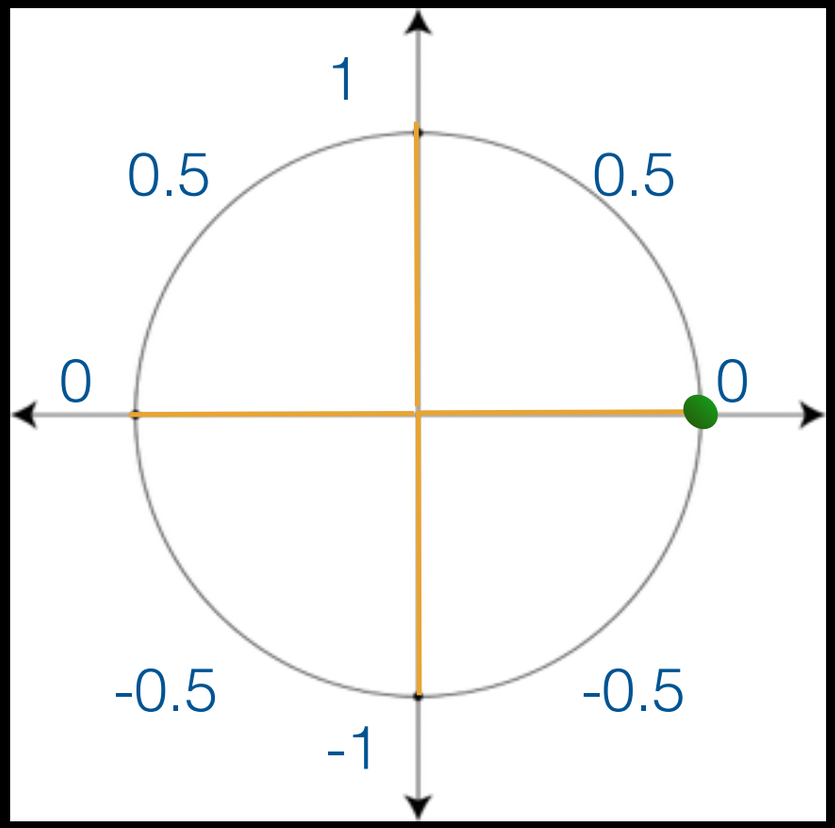
Almost anything that has computer system!

- smart phones
    - sample your voice
    - sample your location
    - sample the temperature
    - sample the light levels to determine screen brightness
- cars
    - ABS systems sample tire rotation
    - sensors sample if someone is behind you
    - dashboard indicators use samples from your engine
- home automation
    - air conditioning samples the temperature in the room

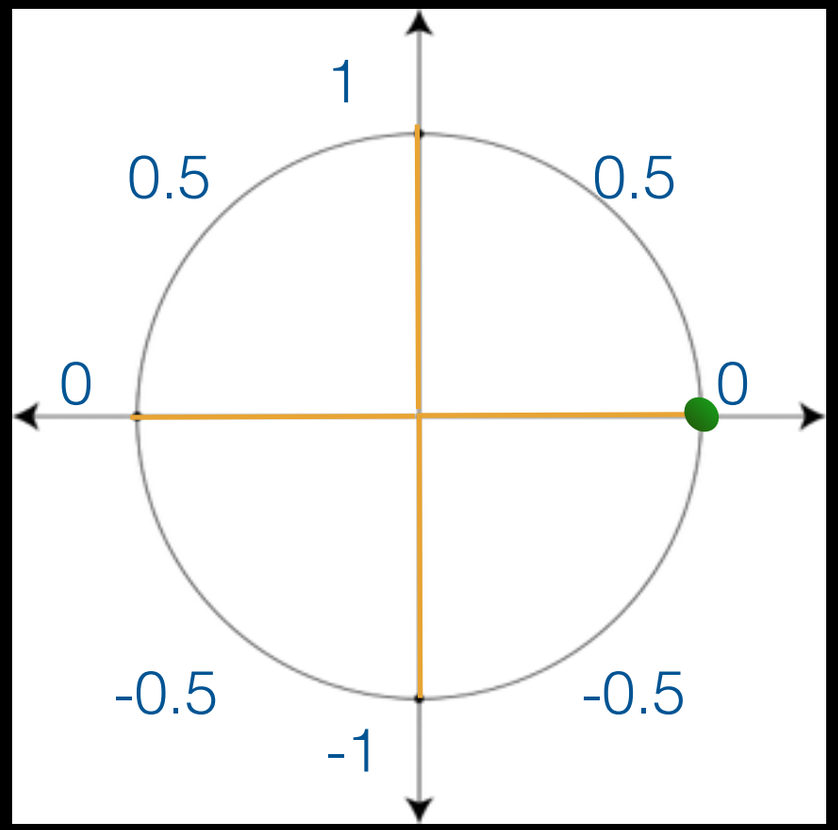
humans!

- the human eye is believed to have a sampling rate!

Let’s talk about things that rotate



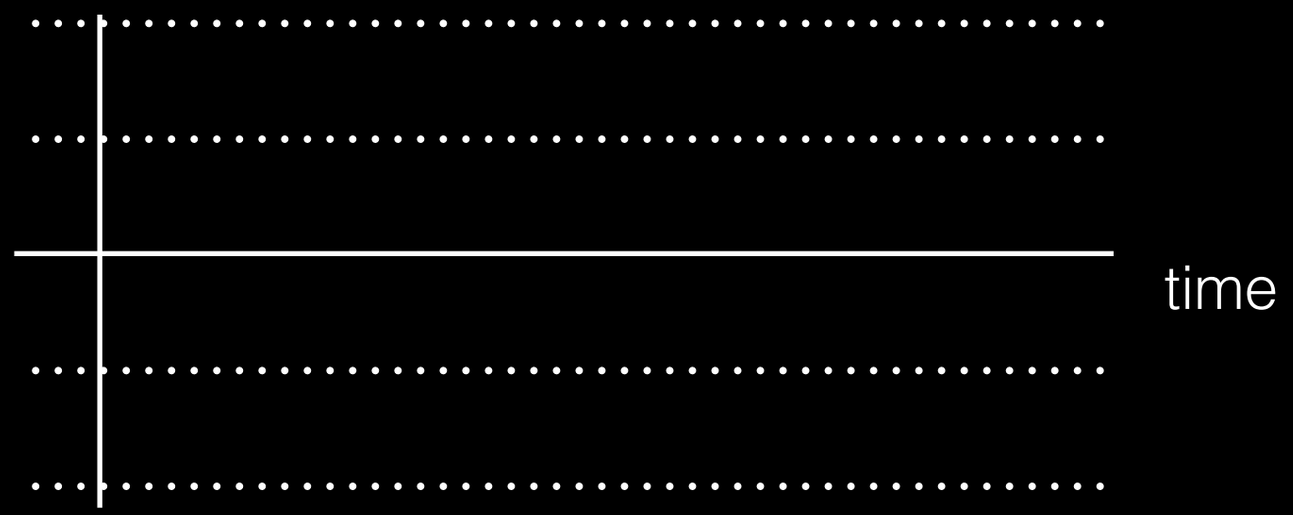
Rotation as a math function



$S(t) = A\sin(\omega t+\phi)$

$S_k = A\sin(\omega(kT)+\phi)$

clockwise

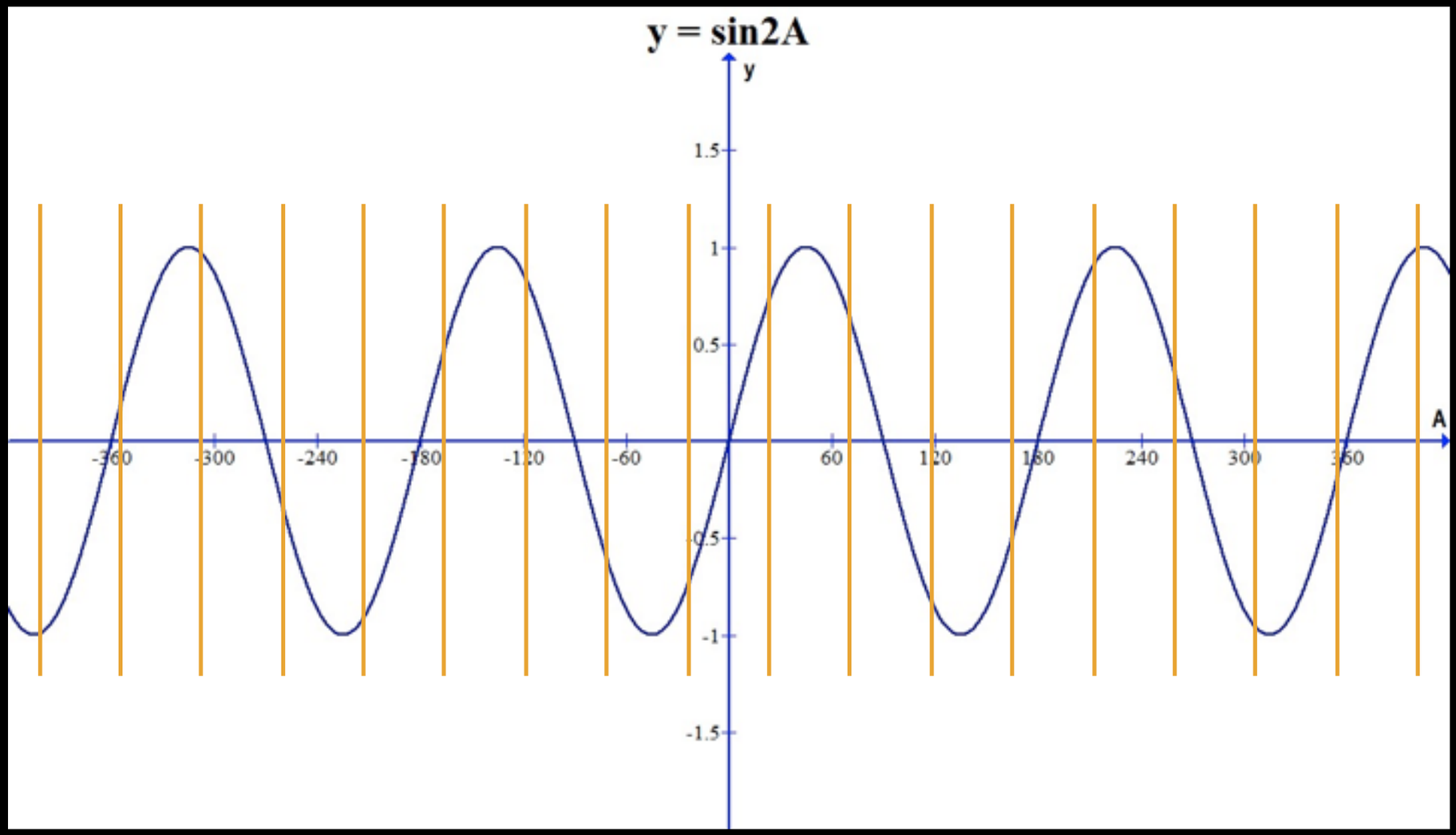


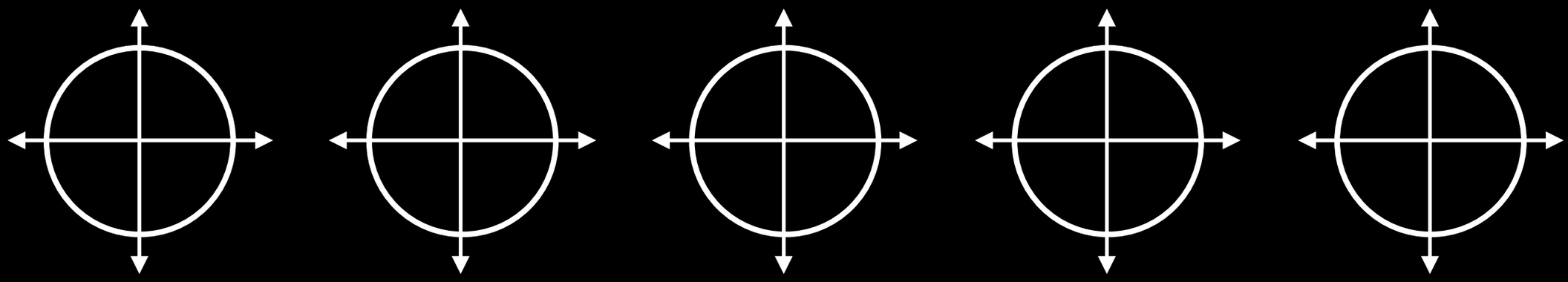
counter clockwise

Sampling things that rotate

Sampling things that rotate

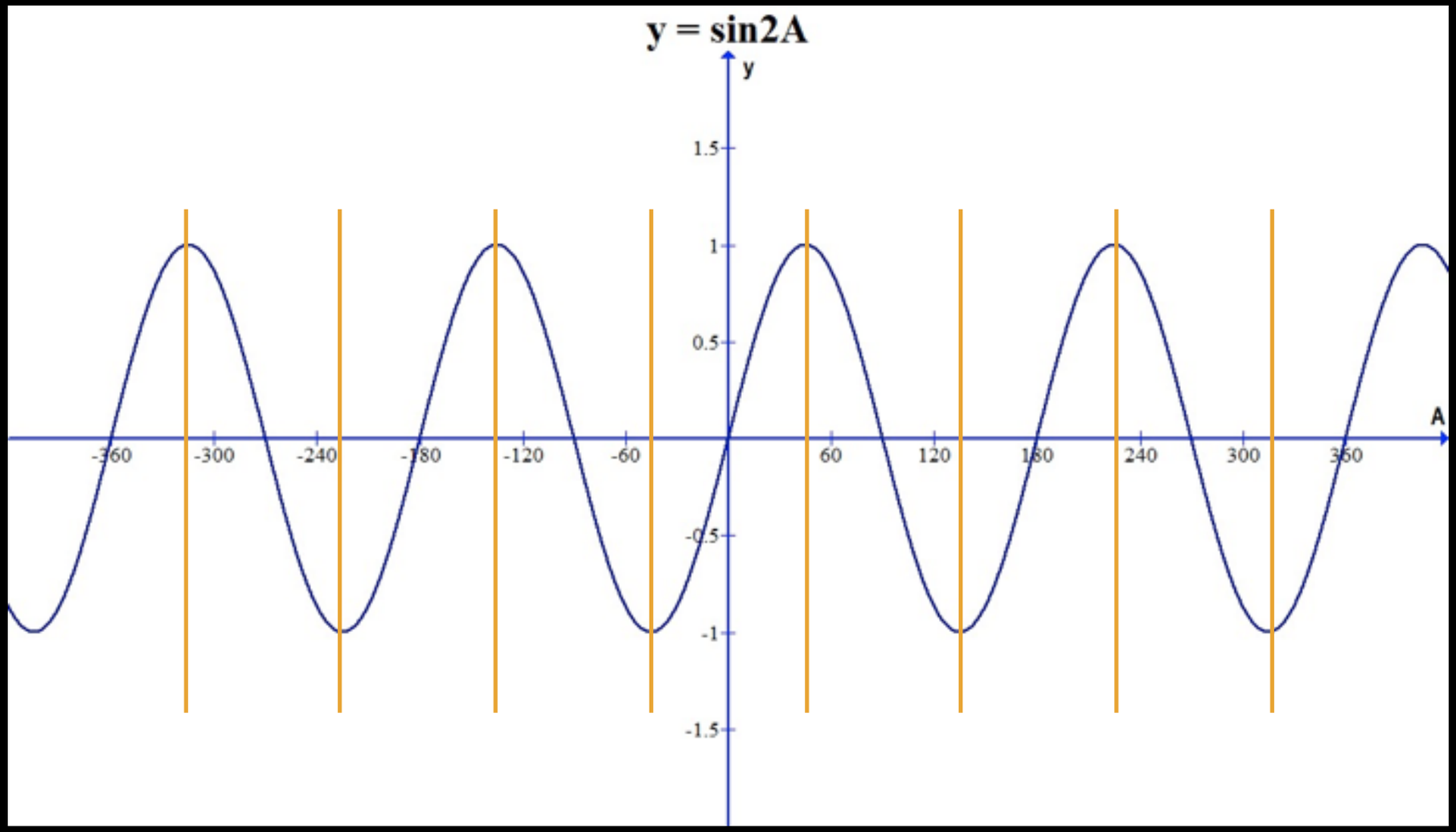
ω=2
1/T=8

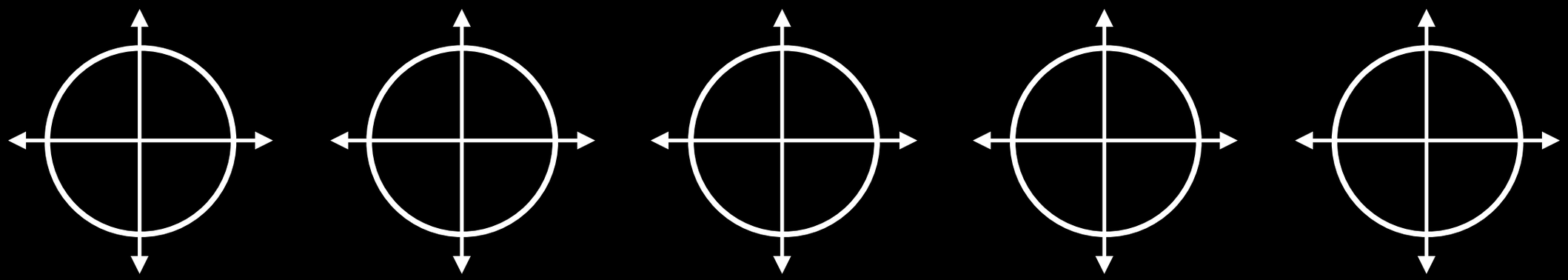




Sampling things that rotate

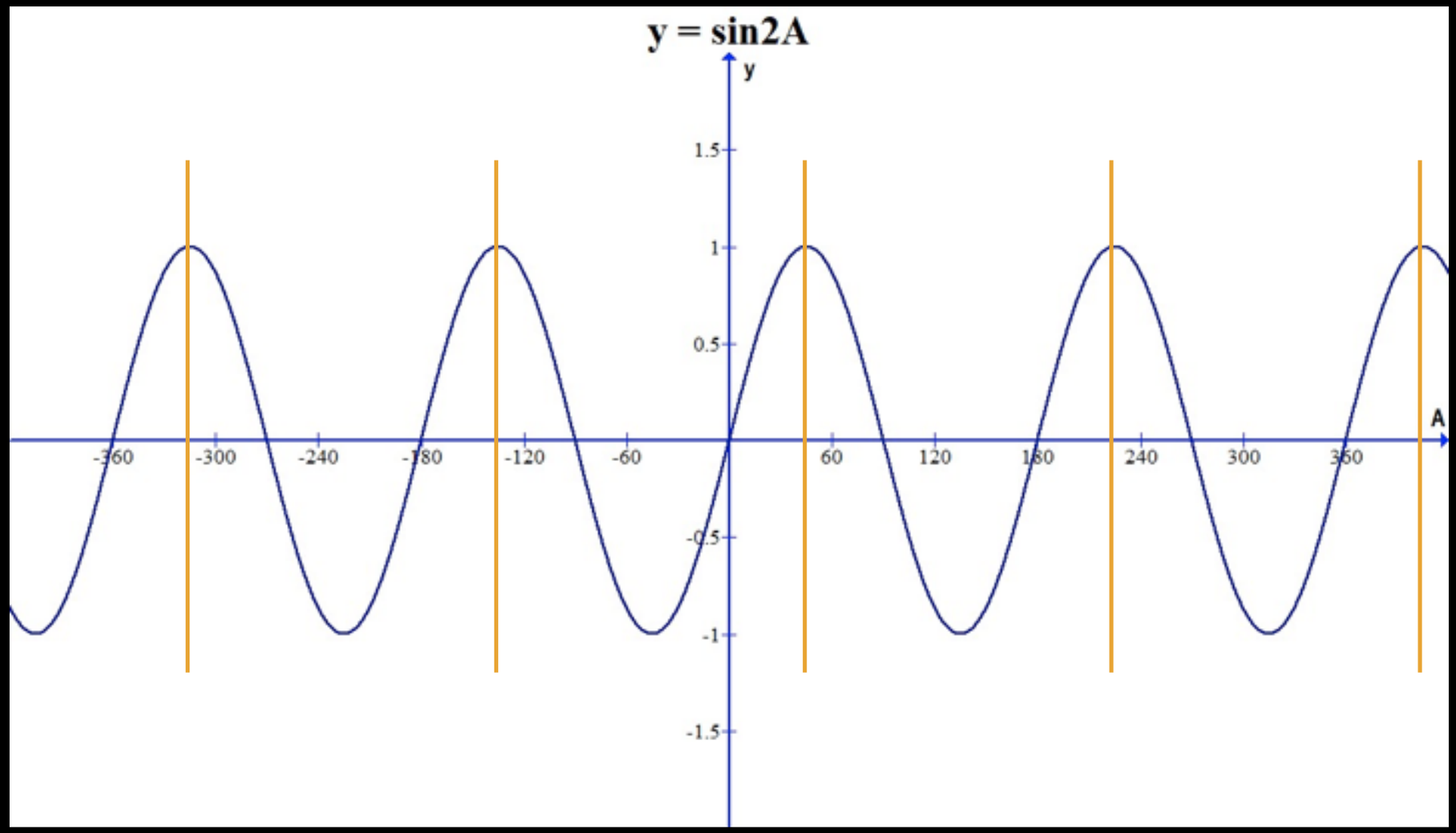
ω=2
1/T=4

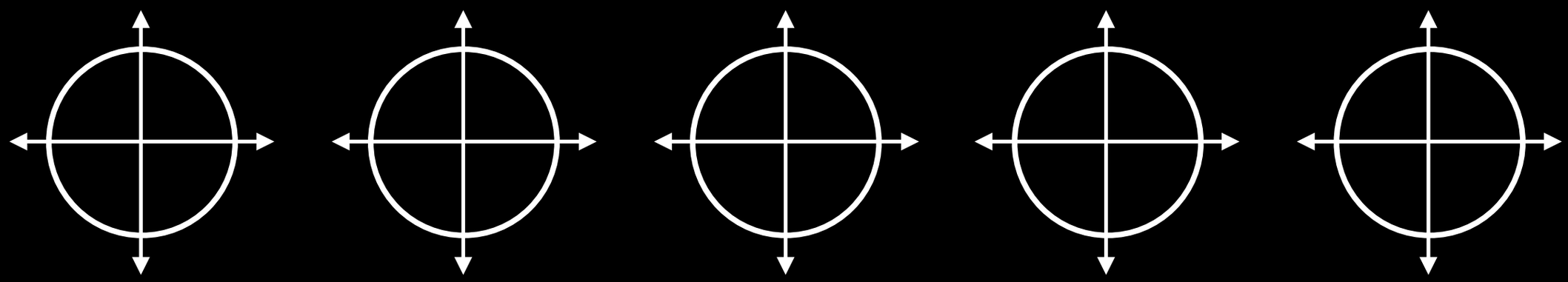




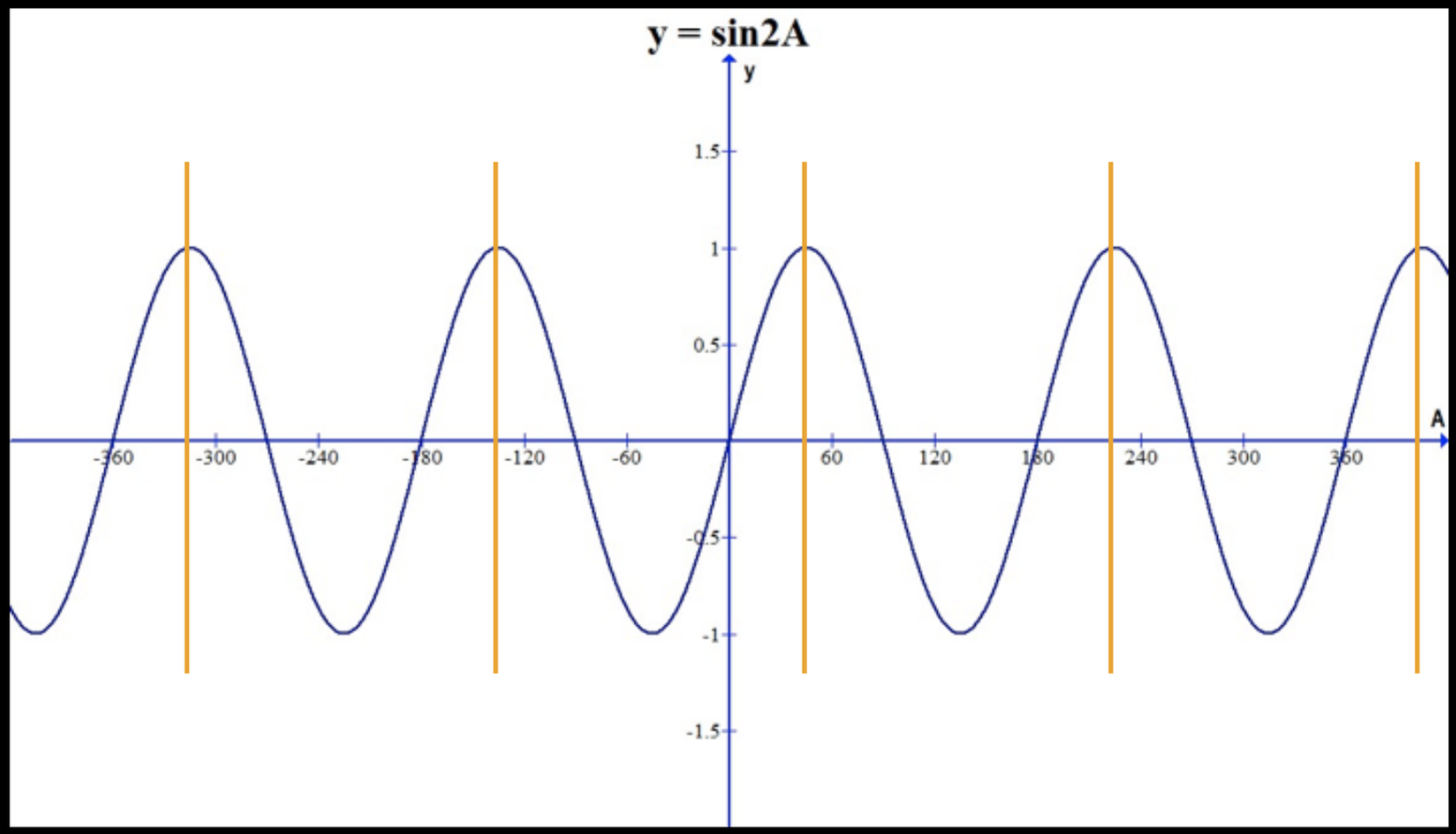
Sampling things that rotate

ω=2
1/T=2





Sampling things that rotate

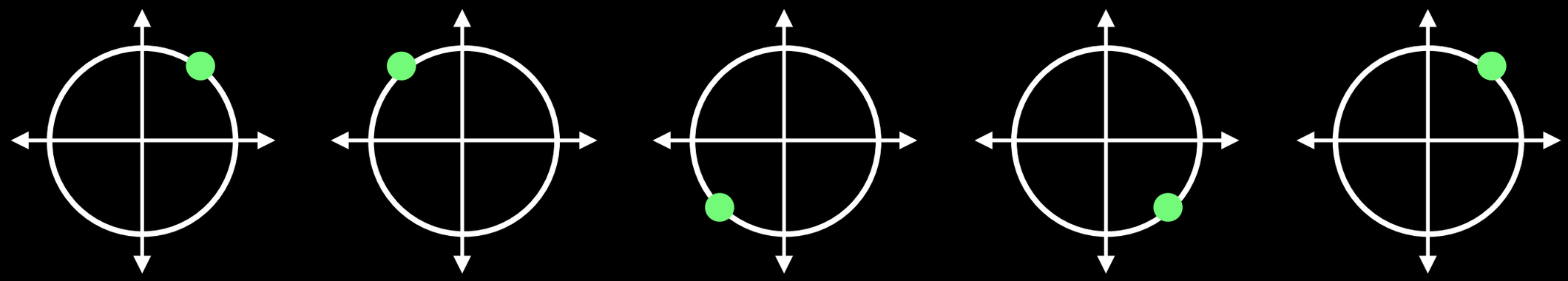


This explains what is happening with the helicopter!

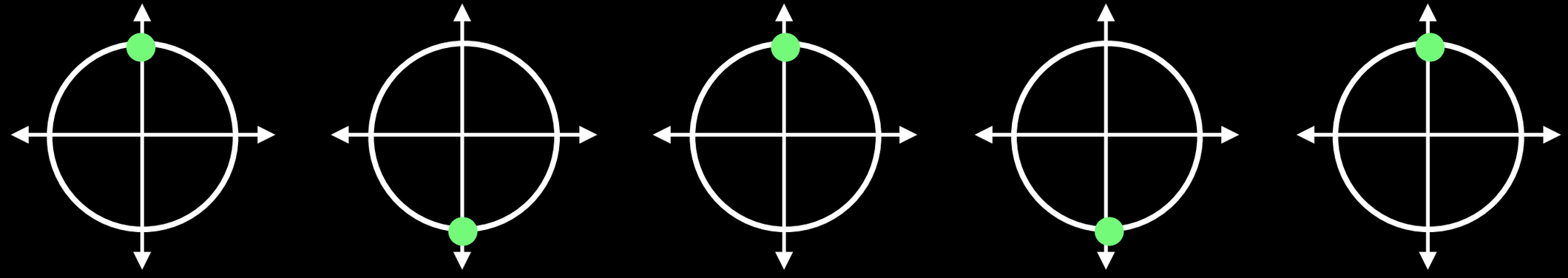
The camera is sampling at a multiple of the rate that the blades are spinning, so the blades look as if they are not moving at all

Summary

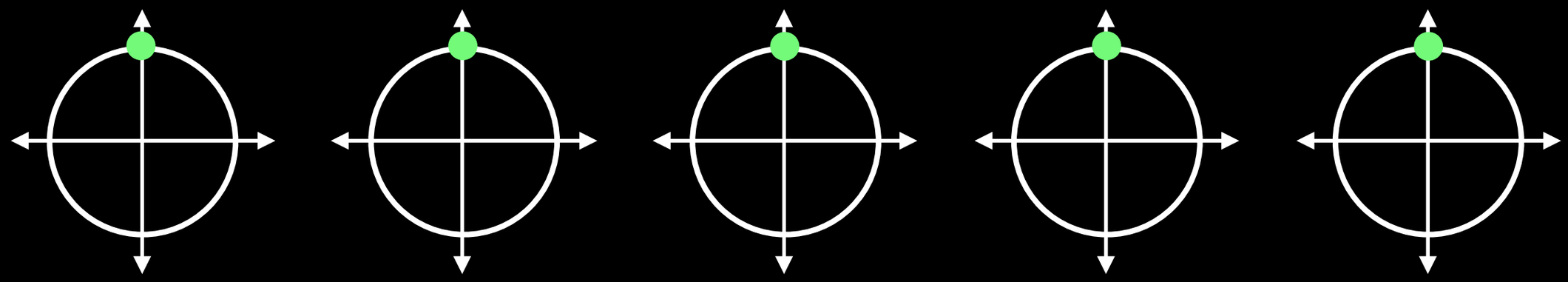
ω=2
1/T=8



ω=2
1/T=4

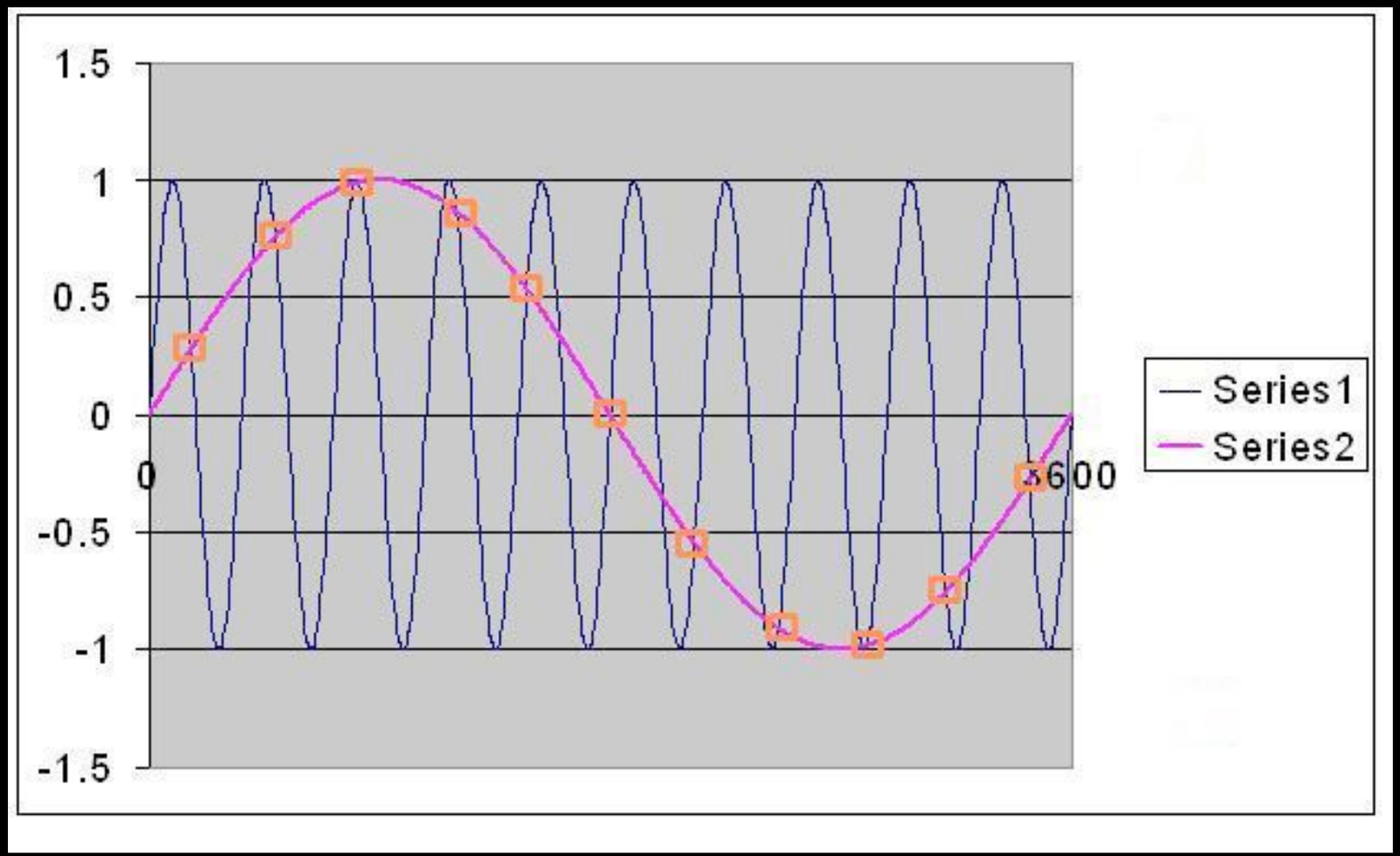


ω=2
1/T=2



What is aliasing?

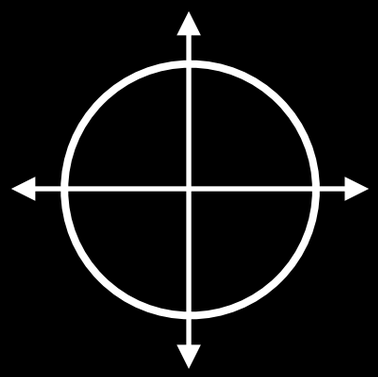
Aliasing happens when we measure things that aren't really there by sampling too slow

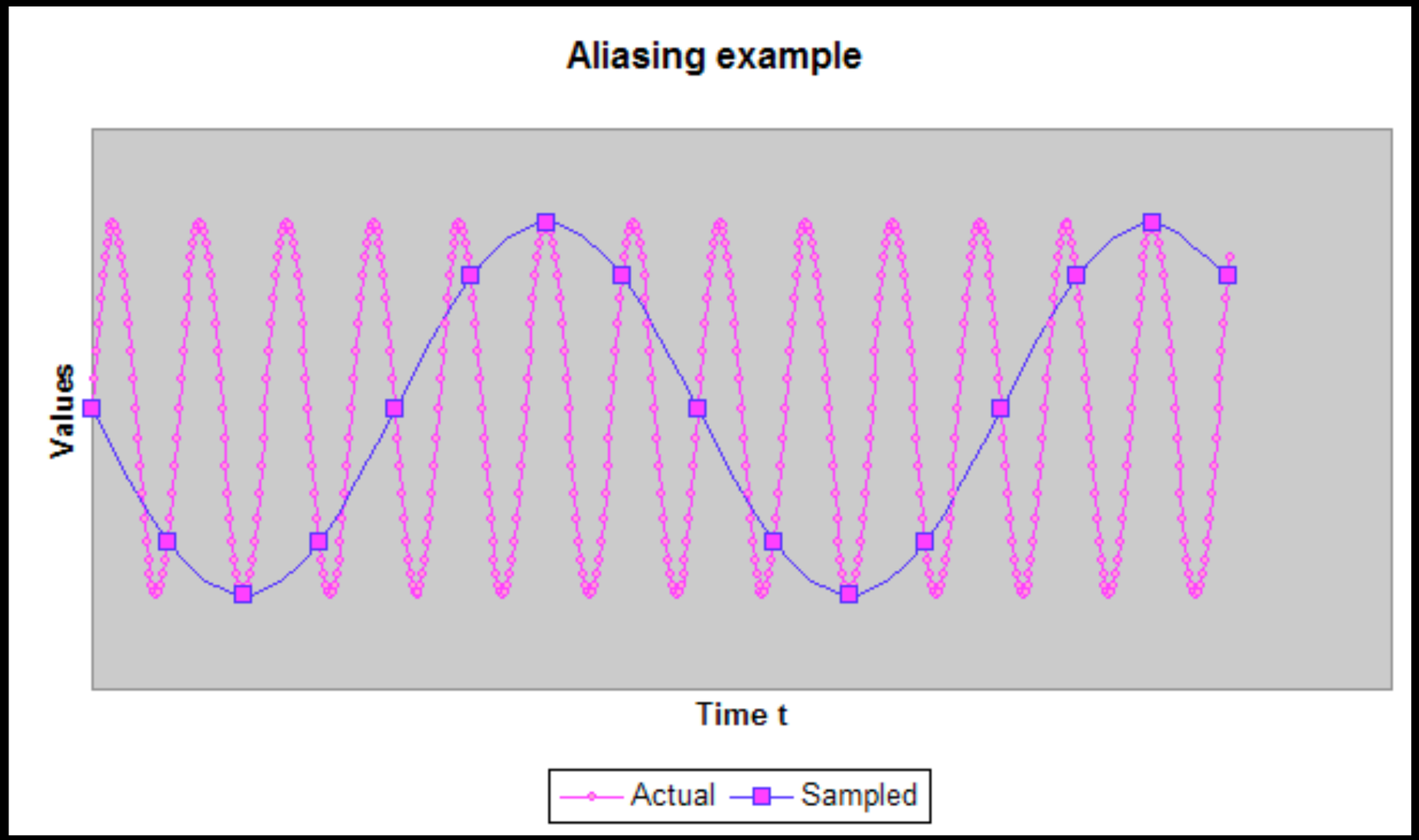


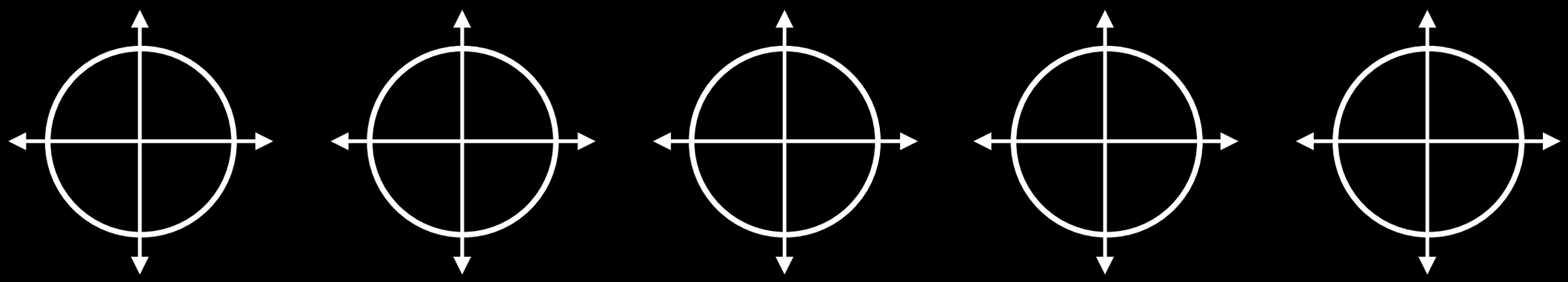
What is happening in this video?

Play
https://www.youtube.com/watch?v=zCSIZ3RflMs

wheels spinning backwards







To Bring it all together…

https://www.youtube.com/watch?v=-Di-nAgwERk

Summary

What we discussed…

- cameras and how they work
- sampling, what is it?
- rotation as a function
- sampling things that rotate
- aliasing, what is it?
- how aliasing explains interesting observations

Enjoy today's class? Tell us about it! We would love to hear your feedback and you can give it to us at:

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