Decentralized Static Output-Feedback Control via Networked Communicaiton

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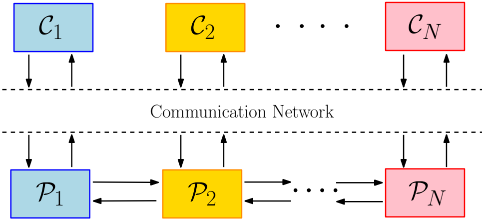
Where innovation starts

Problem Description

We consider stabilizing decentralized control ***design*** for

- large-scale continuous-time linear plant
- via a multi-purpose network with
  - shared communication: not all outputs and inputs can be communicated simultaneously
  - uncertain time-varying transmission intervals $h_k \in [\underline{h}, \overline{h}]\ \forall k \in \mathbb{N}$



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Outline

- NCS Model
  - Disjoint Decomposition
  - Network Effects
  - Closed Loop Model
- Design
  - Overapproximation
  - Multi-gain
- Numerical Example
- Conclusions

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Model - Disjoint Decomposition

The plant is given by a set of disjoint subsystems:

$$\mathcal{P}_i : \begin{cases} \dot{x}_i(t) &= A_i x_i(t) + B_i \hat{u}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(A_{i,j} x_j(t) + B_{i,j} \hat{u}_j(t) \right), \\ y_i(t) &= C_i x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_{i,j} x_j(t), \end{cases}$$

which can be written as

$$\mathcal{P} : \begin{cases} \dot{x}(t) = \underbrace{\begin{bmatrix} A_1 & \cdots & A_{1,N} \\ \vdots & \ddots & \vdots \\ A_{N,1} & \cdots & A_N \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & \cdots & B_{1,N} \\ \vdots & \ddots & \vdots \\ B_{N,1} & \cdots & B_N \end{bmatrix}}_{B} \begin{bmatrix} \hat{u}_1(t) \\ \vdots \\ \hat{u}_N(t) \end{bmatrix}, \\ y(t) = \underbrace{\begin{bmatrix} C_1 & \cdots & C_{1,N} \\ \vdots & \ddots & \vdots \\ C_{N,1} & \cdots & C_N \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix}, \end{cases}$$

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Model - Time-varying Sampling

The set of subsystems can be expressed as

$$\mathcal{P}(t) := \begin{cases} \dot{x}(t) & = & Ax(t) + B\hat{u}(t) \\ y(t) & = & Cx(t) \end{cases}$$

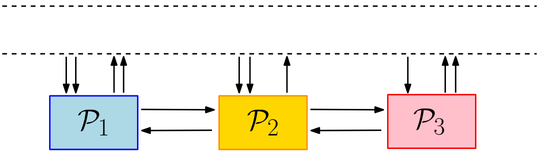
We can express this system with time-varying transmission intervals in the following way

$$\mathcal{P}_{h_k} := \begin{cases} x_{k+1} & = & \bar{A}_{h_k} x_k + \bar{B}_{h_k} \hat{u}_k \\ y_k & = & Cx_k \end{cases}, \quad h_k \in [\underline{h}, \overline{h}]$$

where $\bar{A}_{h_k} := e^{A h_k}$, $\bar{B}_{h_k} := \int_0^{h_k} e^{As} ds B$

* Time-varying delays also can be incorporated similarly as an additive uncertainty

Model - Shared Communication Medium



Model - Shared Communication Medium

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Shared Communication Medium

Only one node is allowed to transmit information at each transmission time

Node - A collection of sensors and/or actuators are allowed communicate over a network simultaneously

Periodic Protocol - grant network access to each node in a periodic fashion

$\sigma_k \in \{1, 2, ..., \bar{N}\}$ denotes the node that has access at transmission time $k \in \mathbb{N}$

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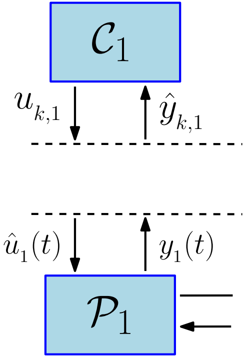
Shared Communication Medium

$$\hat{u}_{j,k} = \begin{cases} u_{j,k} & \text{if node } j \text{ has access} \\ \hat{u}_{j,k-1} & \text{otherwise} \end{cases}$$



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Shared Communication Medium

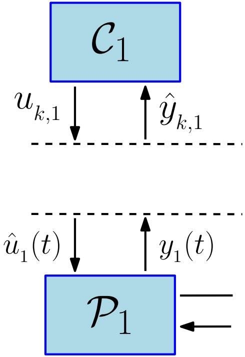
$$\hat{u}_{j,k} = \begin{cases} u_{j,k} & \text{if node } j \text{ has access} \\ \hat{u}_{j,k-1} & \text{otherwise} \end{cases}$$

Mathematically we express this as

$$\begin{bmatrix} \hat{u}_k \\ \hat{y}_k \end{bmatrix} = \Gamma_{\sigma_k} \begin{bmatrix} u_k \\ y_k \end{bmatrix} + (I - \Gamma_{\sigma_k}) \begin{bmatrix} \hat{u}_{k-1} \\ \hat{y}_{k-1} \end{bmatrix}$$

where $\Gamma_{\sigma_k} = diag(\gamma_{j,\sigma_k})$

$$\gamma_{i,\sigma_k} = \begin{cases} 1 & \text{if } u_{j,k}/y_{j,k} \text{ has network access} \\ 0 & \text{otherwise} \end{cases}$$



Model - Overview

Plant Dynamics:

$$\mathcal{P}_{h_k} := \begin{cases} x_{k+1} &= \bar{A}_{h_k} x_k + \bar{B}_{h_k} \hat{u}_k \\ y_k &= \bar{C} x_k \end{cases} \qquad h_k \in [\underline{h}, \overline{h}]$$

Shared Communication:

$$\begin{cases} \hat{u}_k &= \Gamma^u_{\sigma_k} u_k + (I - \Gamma^u_{\sigma_k}) \hat{u}_{k-1} \\ \hat{y}_k &= \Gamma^y_{\sigma_k} y_k + (I - \Gamma^y_{\sigma_k}) \hat{y}_{k-1} \end{cases} \qquad \sigma_k \in \{1, \ldots, \bar{N}\}$$

Controller:

$$\mathcal{C}_{\sigma_k} : u_k = K_{\sigma_k} \hat{y}_k$$

$$K_{\sigma_k} = \text{diag}(K_{\sigma_k,1}, K_{\sigma_k,2}, \ldots, K_{\sigma_k,N}),$$

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Model - Closed Loop

The closed loop model can be written as a discrete-time switched system with exponential uncertainty:

$$\bar{x}_{k+1} = \tilde{A}_{h_k,\sigma_k}\bar{x}_k, \quad h_k \in [\underline{h}, \overline{h}], \;\; \sigma_k \in \{1, ..., N\}$$

where

$$\bar{x}_k^\top = \begin{bmatrix} x_k & \hat{u}_{k-1} & \hat{y}_{k-1} \end{bmatrix}$$

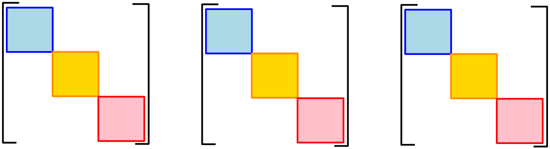
and

$$\tilde{A}_{h_k,\sigma_k} = \left[\begin{array}{c|cc} \bar{A}_{h_k} + \bar{B}_{h_k}\Gamma^u_{\sigma_k}K_{\sigma_k}\Gamma^y_{\sigma_k}C & \bar{B}_{h_k}\Gamma^u_{\sigma_k}K_{\sigma_k}(I-\Gamma^y_{\sigma_k}) & \bar{B}_{h_k}(I-\Gamma^u_{\sigma_k}) \\ \hline \Gamma^y_{\sigma_k}C & (I-\Gamma^y_{\sigma_k}) & 0 \\ \Gamma^u_{\sigma_k}K_{\sigma_k}\Gamma^y_{\sigma_k}C & \Gamma^u_{\sigma_k}K_{\sigma_k}(I-\Gamma^y_{\sigma_k}) & (I-\Gamma^u_{\sigma_k}) \end{array}\right]$$

Structural Constraints

$K_l =$

'Classical'
Decentralized Structre



$l = 1$ $l = 2$ $l = 3$

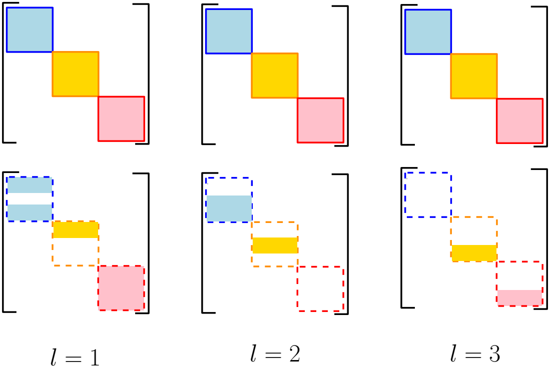
Structural Constraints

$$K_l =$$

'Classical'
Decentralized Structre

$$\Gamma^u_l K_l =$$

Decentralized Structre
with Shared Communication



-> Time-varying structural constraints

Design

Design Problem:

Given a decomposition, a protocol, and $[\underline{h}, \overline{h}]$, how to choose $\Gamma^u_{\sigma_k} K_{\sigma_k}$ such that the closed-loop NCS is globally exponentially stable?

Goal:

Provide LMI conditions to design $\Gamma^u_{\sigma_k} K_{\sigma_k}$ using the Lyapunov candidate

$$V_{\sigma_k}(x_k) = \bar{x}_k^\top P_{\sigma_k} \bar{x}_k > 0, \quad \bar{x}_k \neq 0$$

which must satisfy

$$\Delta V_{\sigma_k}(x_k) = \bar{x}_k^\top (\tilde{A}^\top_{h_k,\sigma_k} P_{\sigma_{k+1}} \tilde{A}_{h_k,\sigma_k} - P_{\sigma_k}) \bar{x}_k < 0, \quad \bar{x}_k \neq 0$$

Design

Challenges:

1. Uncertain nonlinearity
2. Time-varying structural constraints

Our Solution:

1. Create a convex overapproximation of the closed-loop system
2. Rewrite the constrained matrices as a multi-gain output feedback
3. Combine above two techniques to form sufficient LMI synthesis conditions

Design - Overapproximation

Due to the transmission variance $h_k \in [\underline{h}, \overline{h}] \; \forall k \in \mathbb{N}$, there is an infinite amount of sequences to check for guaranteeing stability

$$\left\{ \tilde{A}_{h,\sigma} \mid h \in [\underline{h}, \overline{h}] \right\}$$

Design - Overapproximation

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$$\left\{\tilde{A}_{h,\sigma} \mid h \in [\underline{h}, \overline{h}]\right\} \subseteq \left\{\sum_{j=1}^{M} \alpha^j \left(F_{\sigma,j} + G_j \Delta H_\sigma\right)\right\}$$

Therefore we synthesize controllers with an overapproximation of the original model, which is achieved by

(i) gridding a finite number of points in $[\underline{h}, \overline{h}]$
(ii) adding norm-bounded uncertainty to each grid point to capture the non-linearity between grid points.

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Therefore we synthesize controllers with an overapproximation of the original model, which is achieved by

(i) gridding a finite number of points in $[\underline{h}, \overline{h}]$
(ii) adding norm-bounded uncertainty to each grid point to capture the non-linearity between grid points.

- + introduces arbitrarily little conservatism
- + direct control over the complexity of the overapproximation

[1] Donkers, et. al. Trans. Autom. Control 2011

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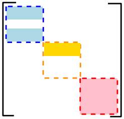
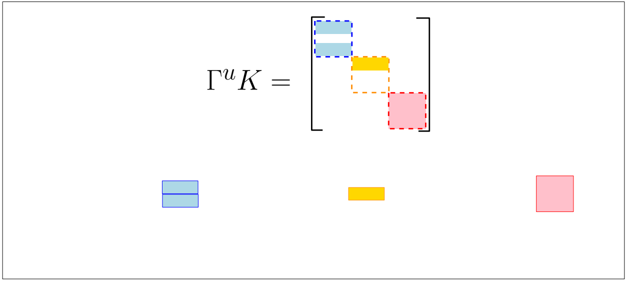
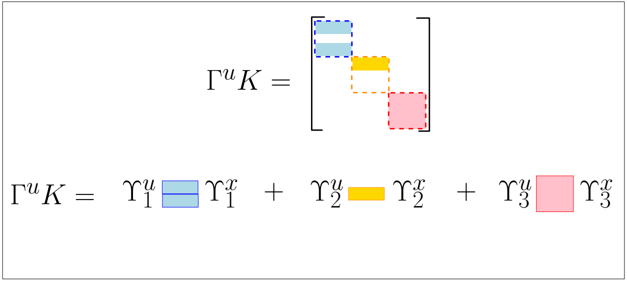
Design - Multi-gain

$$\Gamma^u K =$$




$\Gamma^u K =$

Design - Multi-gain



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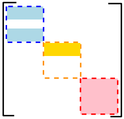
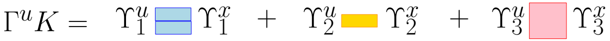
Design - Multi-gain

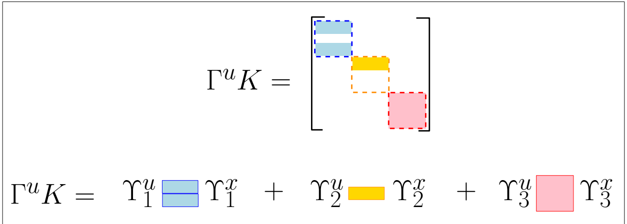
$$\Gamma^u K =$$





$$\Gamma^u K = \Upsilon_1^u \, \bar{K}_1 \, \Upsilon_1^x + \Upsilon_2^u \, \bar{K}_2 \, \Upsilon_2^x + \Upsilon_3^u \, \bar{K}_3 \, \Upsilon_3^x$$

Design - Multi-gain



$$\Gamma^u K = \Upsilon_1^u \bar{K}_1 \Upsilon_1^x + \Upsilon_2^u \bar{K}_2 \Upsilon_2^x + \Upsilon_3^u \bar{K}_3 \Upsilon_3^x$$

$$\Gamma^u K = \sum_{i=1}^{N} \Upsilon_i^u \bar{K}_i \Upsilon_i^x$$

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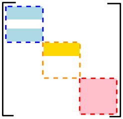
Design - Multi-gain

$\Gamma^u K =$ 

$\Gamma^u K = \Upsilon_1^u$  $\Upsilon_1^x + \Upsilon_2^u$  $\Upsilon_2^x + \Upsilon_3^u$  Υ_3^x

$$\Gamma^u K = \Upsilon_1^u \bar{K}_1 \Upsilon_1^x + \Upsilon_2^u \bar{K}_2 \Upsilon_2^x + \Upsilon_3^u \bar{K}_3 \Upsilon_3^x$$

$$\Gamma^u K = \sum_{i=1}^{N} \Upsilon_i^u \bar{K}_i \Upsilon_i^x$$

$$\bar{x}_{k+1} = \left[\hat{A}_{h_k,\sigma_k} + \sum_{i=1}^{N} (\hat{B}_{h_k,\sigma_k,i} \bar{K}_{\sigma_k,i} \hat{E}_{\sigma_k,i}) \right] \bar{x}_k$$

Design - Theorem Summary

If there exist matrices such that

1. For $j \in \{1, \ldots, \tilde{N}\}$, $m \in \{1, \ldots, M\}$

$$\left[\begin{array}{cc|cc} G_{\sigma_j} + G_{\sigma_j}^\top - P_j & \Xi_1(j,m)^\top & 0 & \Xi_2(j)^\top \\ \star & P_{j+1} & \mathcal{G}_m R_{j,m} & 0 \\ \hline \star & \star & R_{j,m} & 0 \\ \star & \star & \star & R_{j,m} \end{array}\right] \succ 0$$

$$\Xi_1(j,m) := \mathcal{A}_{\sigma_j,m} G_{\sigma_j} + \sum_{i=1}^{N} \mathcal{B}_{\sigma_j,m,i} Z_{\sigma_j,i} \mathcal{C}_{\sigma_j,i}$$

$$\Xi_2(j) := \mathcal{D}_{\sigma_j} G_{\sigma_j} + \sum_{i=1}^{N} \mathcal{E}_{\sigma_j,i} Z_{\sigma_j,i} \mathcal{C}_{\sigma_j,i}$$

2. For $l \in \mathrm{L}_{y,i}$, $i \in \{1, \ldots, N\}$

$$X_{l,i} \mathcal{C}_{l,i} = \mathcal{C}_{l,i} G_l$$

then $\bar{K}_{l,i} = Z_{l,i} X_{l,i}^{-1}$ renders the closed-loop GES.

Numerical Example

Plant Model

	states	inputs	outputs
Subsystem 1:	3	1	2
Subsystem 2:	4	2	2
Subsystem 3:	3	1	2
	10	4	6

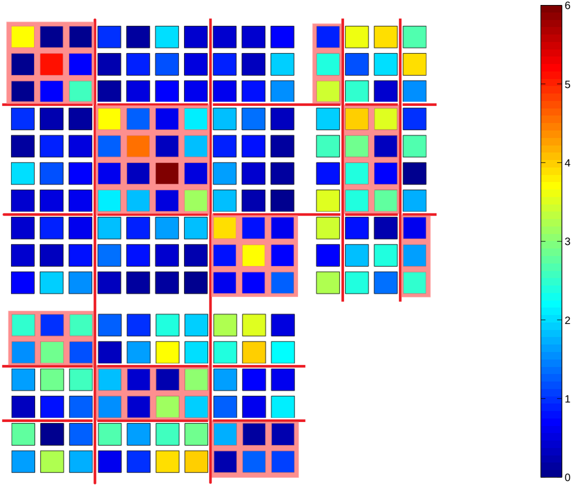
Communication Protocol (when shared):

$$\Gamma_1^u = diag(1, 1, 0, 0), \qquad \Gamma_2^u = diag(0, 0, 1, 1),$$
$$\Gamma_1^y = diag(1, 1, 1, 0, 0, 0), \qquad \Gamma_2^y = diag(0, 0, 0, 1, 1, 1),$$

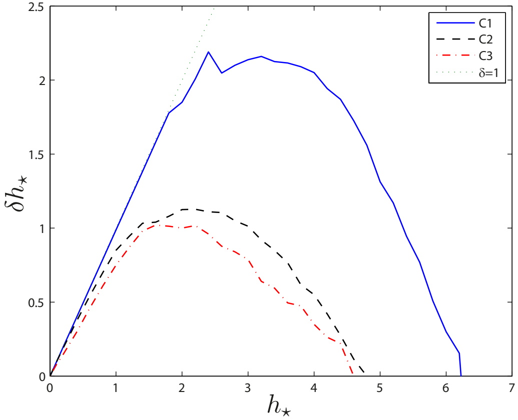
$$\sigma_k = 1, 2, 1, \ldots.$$

Numerical Example

Coupling Visualization



Numerical Example



C1 - Centralized,
Comm. not shared

C2 - Decentralized,
Comm. not shared

C3 - Decentralized,
Comm. is shared

$$h_k \in [(1-\delta)h_\star, (1+\delta)h_\star]$$

Average computation time to solve LMI:
C1, C2: 15 seconds
C3: 40 seconds

Conclusions

- We presented a model for an NCS which includes
  - varying transmission intervals $[\underline{h}, \overline{h}]$
  - shared communication medium (protocol)

  The controllers are
  - static
  - decentralized
  - switch based on protocol
- Sufficient conditions for controller synthesis were provided for the
  - decentralized problem setting
  - NCS problem setting
  - unification of the above two problem settings

  + Single-shot design (‘a posteriori’ analysis not required)
  + No structure was imposed on the Lyapunov function
  + Extendable to synthesizing decentralized observer-based controllers